Basic and Commercial Mathematics

For Competitive Examinations
and Academic Studies

P. O. LUISE

BASIC AND COMMERCIAL MATHEMATICS

**For Competitive Examinations
and Academic Studies**

P. O. LUISE B.Com., MBA, FCMA

Part-I

BASIC MATHEMATICS

*Dedicated to
Our Lady of Perpetual Succour*

Basic and Commercial Mathematics

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FOREWORD TO FIRST EDITION

The book, 'Basic and Commercial Mathematics is an earnest outcome of the experience realised by the author to jump over the mathematical hurdle for a course like I.C.W.A. The topics covered are those of the school level, which ought to have been thoroughly understood by an average student. In fact any graduate or post-graduate of Arts or Commerce find it extremely difficult to follow even the elementary ideas in mathematics which have become part and parcel of every day use. With the result that a student who prepares for any clerical examination, bank test, test in railways, entrance examination for M.B.A., M.C.A. etc. find it very hard to unravel the mysteries embedded in the elementary mathematics of which he had been a student in his teenages. Hence he approaches some coaching centres for the purpose.

This handlook fills up the gap helpfully enabling the aspirant to equip himself well without the aid of a coaching class. I wish the author's endeavour every success.

sd/-
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PREFACE

Now a days almost all the competitive examinations include Mathematics in the syllabus, which makes the examination difficult for Arts and Commerce students. Knowledge of the basic principles of mathematics has, thus become inevitable for all those who are aspiring to take up any competitive examination. Obtaining a clear grasp of the fundamentals of mathematics is an uphill task for an average student.

Keeping in mind the above requirements, sincere attempt has been made to bring out a comprehensive compilation of almost all the chapters consisting of the basic principles, formulae, examples and solutions for numerous problems compiled from the question papers of competitive examinations as well as university examinations. Solutions to all the problems are given immediately thereafter to facilitate ready reference. But the students should try to solve each problem themselves and check their workings with the given solution. The aim of the book is to help the users to understand thoroughly the basic principles and applications of mathematical techniques. It can also be used as a stepping stone towards the advanced level. Considering the requirement of Accountants, Mathematical application in M.S. Excel etc. are included in this revised edition.

I am grateful to my professional friends and colleagues who have rendered valuable assistance and suggestions in the creation of this book.

Any suggestions for improvement of the book will be most gratefully accepted and highly appreciated.

Tripunithura **Author**

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(Part II is not given in this file. It is available in printed book)

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1. FUNDAMENTALS OF MATHEMATICS

A. NUMBERS

Following are the different types of Numbers

1. Counting Numbers or Natural Numbers
(എണ്ണൽ സംഖ്യകൾ അഥവാ നിസർഗ്ഗ സംഖ്യകൾ)
1, 2, 3, 4, 5, 6, 7,.....
2. Whole Numbers (അഖണ്ഡ സംഖ്യകൾ)
0, 1, 2, 3, 4, 5,.....
3. Odd Numbers (ഒറ്റ സംഖ്യകൾ)
1, 3, 5, 7, 9, 11, ...
4. Even Numbers (ഇരട്ട സംഖ്യകൾ)
2, 4, 6, 8, 10,12,.....
5. Integers (പൂർണ്ണ സംഖ്യകൾ)
.....-5, -4, -3, -2, -1, 0, +1, +2, +3, +4,.....
6. Fractions (ഭിന്ന സംഖ്യകൾ)
Ex: $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{7}{6}, 1\frac{1}{4}$ etc
7. Decimal Numbers (ദശാംശസംഖ്യകൾ)
Ex: 0.5, 0.15, 0.001, 3.12, etc.

8. Terminating Decimals

Ex:- $\frac{10}{4} = 2.5$, $\frac{1}{8} = 0.125$, etc

9. Non- Terminating Decimals

Ex:- $\frac{10}{3} = 3.3333.....$ $\frac{2}{3} = 0.66666.....$

10. Recurring decimals:

Ex:- $\frac{1}{3} = 0.33333.....$, $\frac{2}{11} = 0.18181818.....$

11. Non -recurring decimals

Ex:- $\frac{1}{7} = 0.142857.....$, $\frac{3}{13} = 0.23076.....$

12. Rational Numbers:

A number which can be written in the form $\frac{p}{q}$, where p & q are integers and $q \neq 0$., is called a rational number.

Ex: 3, -2, $\frac{1}{8}$, $\frac{1}{3}$, etc. $\left\{ 3 = \frac{6}{2} = \frac{9}{3}, \quad -2 = \frac{-4}{2} = \frac{-6}{3} \right\}$ $\left\{ \frac{1}{8} = 0.125, \quad \frac{1}{3} = 0.3333..... \right\}$

13. Irrational Numbers:

A non- terminating and Non- recurring decimal is an irrational number.

Ex: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc. [$\sqrt{2} = 1.4142...$, $\sqrt{3} = 1.732....$]

14. Surds (കരണികൾ)

Ex:- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$.....

15. Ascending order of Numbers (ആരോഹണക്രമം)

Ex:- 1, 2, 3, 4, 5, 6, ...

16. Descending order of Numbers (അവരോഹണക്രമം)

Ex:- 100, 99, 98, 97, 96.....

17. Prime numbers (അഭാജ്യസംഖ്യകൾ)

If a number has only two factors, it is called a prime number.

Ex:- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29,.....

18. Composite numbers:-

If a number has more than two factors, It is called a composite number

Ex:- 4, 6, 8, 9, 10,12, 14, 15.....

The number '1' is neither prime nor composite, because it has only one factor.

B. READING AND WRITING OF NUMBERS

Indian System of Numeration

Our system of numeration is called Hindu- Arabic , base-10 or the decimal system. It uses only ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9 called digits. Every whole number can be written with the help of these symbols.

Expanded from of 56312 is 50000 + 6000+ 300+ 10+ 2

The Number 17283546 reads as

"One crore, seventy two lakh, eighty three thousand, five hundred and forty-six' [1,72,83,546]

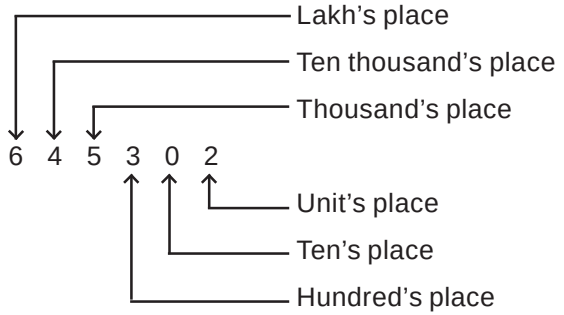
Face Value and Place Value of Numbers

Face Value of a Digit is the digit itself.

Ex:- Face Value of 3 in the number 4305 is 3.

Place value varies depending on the place of digit in a number

Ex:- Place value of three in the number 4305 is 300



Ex:- Find the difference of the place value and the face value of the digit 4 in 34,58,052 ? [L. S. S. Scholarship Exam]

Ans : Place Value = 4,00,000
Face Value = 4
Difference = 4,00,000 - 4 = **3,99,996**

International System of Numeration

In this system the number 17283546 reads as ' Seventeen million, two hundred -eighty-three thousand, five hundred and forty six' [17,283,546]

1 million = 10 lakhs
1 billion = 1000 million
1 billion = 100 crores

(In british system of numeration, a billion equals one million millions)

C. THE FOUR RULES OF ARITHMETIC

Arithmetic is the study of numbers 1, 2, 3, 4,.....under various operations of which the simplest are addition, subtraction, multiplication and division . These are so-called 'Four Rules'. The word 'Arithmetic' is derived from the Greek word " arithomos" meaning number.

Speed and accuracy in simple calculations must first be mastered, and the student is advised to work as many examples as possible.

1. Addition (സങ്കലനം) (+)

Joining of two or more similar numbers is called addition. The symbol for addition is '+' (plus) from Latin, meaning more.

Ex: Find the sum of the largest and the smallest number of five digits?

Ans: Largest number of 5 digits = 99999
Smallest number of 5 digits = 10000
Sum = 99999+10000
= **109999**

`

2. Subtraction (വ്യവകലനം) (-)

It is the opposite of addition. The symbol for subtraction is '-' (minus), from the Latin, meaning less; it is placed between two numbers, when the second is to be taken away from the first.

Ex: What is the difference between the greatest and smallest number that can be formed with the digits 8, 2, 5, 0 and 1?

Ans: Greatest number = 85210
Smallest number = 10258
Difference = 85210 - 10258
= **74952**

85210 –
10258
74952

3. Multiplication (ഗുണനം) (x)

Multiplication is the repeated addition of the same number.

Ex:- 1. $6 \times 3 = 6 + 6 + 6 = 18$

2. $20 + 15 \times 310 = ?$ [S.B I. P.O. Exam]

Ans: 20 + 4650 = **4670**

310 ×
15
1550
310
4650

4. Division (ഹരണം) (÷)

Division is repeated Subtraction,

Dividend = Quotient × Divisor + Remainder

Ex: $317 \div 3 = \frac{317}{3} = 105\frac{2}{3}$

105 ← Quotient
Divisor → 3 | 317 ← Dividend
3
17
15
2 ← Remainder

Let x be a number, then

$\frac{x}{x} = 1$	$\frac{0}{x} = 0$
$\frac{x}{1} = x$	$\frac{x}{0}$ is not defined

A number is exactly divisible when there is no remainder after division. Quotient will be a whole number.

Ex: $\frac{10}{2} = 5$, $\frac{15}{3} = 5$, $\frac{24}{4} = 6$

Problems:

1) Write down in figures :
Two Lakhs one thousand and one.
(L.I.C Asst. Grade II Exam)

Ans: 2,01,001

2) Write down in words : 180203003
(P.S.C Divisional Accountant Test)

Ans: Eighteen crores, two lakhs, three thousand and three.

3) 11 times a number gives 132, The number is ___
(Clerk's Grade Exam)

Ans: 132 ÷ 11 = 12

4). The sum of any three consecutive natural numbers is always divisible by _____ (Clerk's Grade Exam)

Ans: 3

5) 111111 ÷ 11 = ... [Bank Clerical Exam (Jaipur)]

Ans: 10101

```
      10101
   ┌──────
11 │111111
   │11
   ──
    11
    11
   ──
     11
     11
   ──
     00
   ──
```


6) How many dollars would equal to Rs. 10,000 if the rate of exchange is 2.5 dollars for Rs. 100 ?

Ans: Rs.10,000 × $\frac{2.5}{100}$ = 250 dollars.

7) On dividing 55390 by 299 , the remainder is 75 then the quotient is _____ (P S C Divisional Accountant Test)

Ans: Quotient : = (Dividend - Remainder) ÷ Divisor
= (55390 – 75) ÷ 299
= 55315 ÷ 299 = 185

D. TESTS FOR DIVISIBILITY OF NUMBERS

I Divisibility by 10:

A number is exactly divisible by 10, if its unit's digit is zero.

Ex:- 10, 20, 40, 110, 3140

II Divisibility by 5:

If the digit in the unit's place of a number is zero or 5, It is exactly divisible by 5

Ex: 5, 10, 15 ,55, 105, 850, 10095

III Divisibility by 2:

If the digit in the unit's place of a number is 0, 2, 4, 6 or 8 (ie, even number or zero), it is exactly divisible by 2.

Ex: 2, 4, 10, 106, 3458, 8500

IV Divisibility by 4:

A number is exactly divisible by 4, if the last two digits are zeros or divisible by 4.

Ex:- 12, 100, 208, 23140, 6796

V Divisibility by 8

A number is divisible by 8, if the last three digits are zeros or divisible by 8.

Ex: 32, 240, 1024, 99096, 58000, 109008

VI **Divisibility by 3:**

If the sum of the digits of a number is divisible by 3, the number is exactly divisible by 3.

Ex: 345, 7002, 10902

(Sum of the digits of 345 = 3+4+5 = 12

12 is divisible by 3. Therefore, 345 is divisible by 3)

VII **Divisibility by 9:**

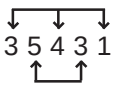
If the sum of the digits of a number is divisible by 9, the number is exactly divisible by 9.

Ex: 18, 81, 108, 90243

VIII **Divisibility by 11:**

A number is exactly divisible by 11, if the difference of the sum of its digit in odd places and the sum of its digit in even places (Starting from unit place) is either zero or divisible by 11.

Ex: 121, 9053, 35431



Sum of digits in odd places from right = 1+ 4 + 3 = 8

Sum of digits in even places from right = 3 + 5 = 8

Difference = 8 - 8 = 0

35431 is divisible by 11.

IX **Divisibility by 6, 15, 45, etc**

A given number is exactly divisible by another number, if the given number is divisible by the factors of the other number.

ie. A number is divisible by 6, if it is divisible by 2 & 3.

A number is divisible by 15, if it is divisible by 3 & 5.

A number is divisible by 45, if it is divisible by 5 & 9.

Problems :

1. The number of 3 digit numbers exactly divisible by 5 is_____.

   (181, 180, 179, 199) [Clerks Grade Exam]

   Ans: **180**

2. If the number 25*84 is divisible by 9, the digit in place of * should be _____.

   [Inspector of central Excise, Income tax etc. Exam]

   Ans: Sum of the digits = 2 + 5 + * + 8 + 4 = 19 + *

   Next multiple of 9 after 19 is 27, * = 27 - 19 = **8**

3. What is the smallest number that should be added to 139 to make it exactly divisible by 3 ?

   Ans: Sum of the digits of 139 = 1 + 3 + 9 = 13

   Next multiple of 3 after 13 = 15

   Number to be added to 139 to make it exactly divisible by 3 } = 15 -13 = **2**

4. If the number 3003* 4 is divisible by 6, then the digit at the place of * is _____. (Clerk's Grade Exam)

   Ans: Sum of the digits = 3 + 0 + 0 + 3 + * + 4 = 10 + *

   Next multiple of 3 after 10, is 12, Answer is 12-10 = **2**

5. If 756* is a multiple of 11, then the * is to be replaced by _____

   (N.D.A. Exam.)

   Ans: x = Sum of 2^{nd} and 4^{th} digits = 6 + 7 = 13

   y = Sum of 1^{st} and 3^{rd} digitd = * + 5

   If the number 756* is divisible by 11, x-y must be zero or a multiple of 11

   To get the difference between x and y is equal to zero, y should be 13.

   $\therefore$ * = 13 - 5 = **8**

E. PROPERTIES OF ADDITION AND MULTIPLICATION

1. Closure Property:

Ex: $5 + 3 = 8$

$5 \times 3 = 15$

2. Commutative property:

Ex: $3 + 4 = 4 + 3$ ie, $a + b = b + a$

$3 \times 4 = 4 \times 3$ ie , $a \times b = b \times a$

3. Associative Property:

Ex: $(10 + 12) + 8 = 10 + (12 + 8)$ ie, $(a + b) + c = a + (b + c)$

$(2 \times 3) \times 5 = 2 \times (3 \times 5)$ ie, $(a \times b) \times c = a \times (b \times c)$

4. Distributive Property :

Ex: $3 \times (5 + 2) = 3 \times 5 + 3 \times 2$

ie: $a \times (b + c) = a \times b + a \times c = ab + ac$

$K(a+b+c) = Ka + Kb + Kc$

5. Existence of identity :

Ex: $8 + 0 = 0 + 8 = 8$ ie, $a + 0 = 0 + a = a$

$7 \times 1 = 1 \times 7 = 7$ ie, $a \times 1 = 1 \times a = a$

'0' is the identity element of addition

'1' is the identity element of multiplication.

6. Existence of Inverse:

Additive inverse of a is (-a) ie, $a + (-a) = 0$

" $\left(\frac{a}{b}\right)$ is $\left(\frac{-a}{b}\right)$ ie, $\left(\frac{a}{b}\right) + \left(\frac{-a}{b}\right) = 0$

Multiplicative inverse of a is $\frac{1}{a}$ ie, $a \times \frac{1}{a} = 1$

" $\left(\frac{a}{b}\right)$ is $\left(\frac{b}{a}\right)$ ie, $\frac{a}{b} \times \frac{b}{a} = 1$

F. SIMPLIFICATION

For Simplification, involving all the four operations along with the bracket sign, the rule "BODMAS" must be followed.

BODMAS		
B	→	Bracket
O	→	Of
D	→	Division
M	→	Multiplication
A	→	Addition
S	→	Subtraction

The rule is first remove the brackets, and then do the works of division , multiplication, addition and subtraction.

Problems:

1 $1.1 + 0.9$ of $9 = ?$

Ans: $1.1 + 0.9 \times 9 = 1.1 + 8.1 =$ **9.2** [of = '×']

2 $3 + 4 \times 3 = 3 + 12 =$ **15**
(Addition is only after multiplication)

3 Solve $8 - [4 + \{ 9 - (12 - 7)\}]$ (M.B.A Entrance Exam)

Ans: $8 - [4 + \{9 - 5\}]$

$8 - [4 + 4] = 8 - 8 =$ **0**

4 $\frac{12 + 8 \times 3 - 5}{4 + 5 \times 2 - 6} = ?$ [Bank P.O Exam]

Ans: $\frac{12 + 24 - 5}{4 + 10 - 6} = \frac{36-5}{14-6} = \frac{31}{8} =$ **$3\frac{7}{8}$**

5 $8 \times 5 \div 5 + 3 = ?$ [R. R. B.E (Bombay)]

Ans: $8 \times 1 + 3 = 8 + 3 =$ **11**

6. $8 - 3[16 - 2\{4 - (6 - 10)\}] = x - 5,\ x = ?$

Ans: $= 8 - 3[16 - 2\{4 - (-4)\}]$

$= 8 - 3[16 - 2\{8\}]$

$= 8 - 3[16 - 16]$

$= 8 - 3 \times 0$

$= 8 - 0$

$= 8$

$x - 5 = 8$

$x = 8 + 5 = \mathbf{13}$

7. $88 - [87 - \{59 - (43 - \overline{29 - 19})\}] = ?$

Ans: $= 88 - [87 - \{59 - (43 - 10)\}]$

$= 88 - (87 - \{59 - 33\})$

$= 88 - (87 - \{59 - 33\}]$

$= 88 - (87 - 26)$

$= 88 - 61$

$= \mathbf{27}$

G. FACTORS AND MULTIPLES

Factors (ഘടകങ്ങൾ) :

A factor of given number is a number which divides the given number exactly.

Ex: Factors of 9 are 1, 3, 9

Factors of 12 are 1, 2, 3, 4, 6, 12

1 is a factor of every number.
Any number is a factor of itself.

Prime Factors (അഭാജ്യ ഘടകങ്ങൾ) :

The prime numbers which are factors of a number are called its prime factors.

Ex: Factors of 30 → 1, 2, 3, 5, 6, 10, 15, 30

Prime factors of 30 → 2, 3, 5

Multiples (ഗുണിതങ്ങൾ) :

A multiple of a number is exactly divisible by the number.

Ex: Multiples of 4 are 4, 8, 12, 16, 20.....

Every number is a factor as well as multiple of itself

Factorisation of a number :

To Factorise a given number, split the number into its prime factors, such that their product is the same number.

Ex:- Factorise 1260

2	1260
2	630
3	315
3	105
5	35
	7

$2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

Prime numbers :

A prime number is a number which is not divisible by any one number except itself and unity.

ie, it has only two factors.

eg : 2, 3, 5, 7, 11, ... (Factors of 5 are 1 and 5)

2 is the only one prime number that is even.

Twin Primes:

Primes occuring in pairs with a difference of two are called twin primes.

Ex: 3 & 5

11 & 13

Co- Prime numbers:

Two numbers are said to be co-prime if they do not have a common factor other than 1.

Ex: 8 and 15 are co- prime numbers .

Factors of 8 = 1, 2, 4, 8

Factors of 15 = 1, 3, 5,15. Common factor is only 1.

If a number is divisible by each of the two or more co-prime numbers, it must be divisible by their product.

Co-prime numbers of 45 are 5 and 9.

A number is divisible by 45, if it is divisible by 5 & 9.

Problems:

1) The number of primes ≤ 30 is ____. (N.D.A Exam)

Ans: 10. (ie. 2, 3, 5, 7, 11, 13, 17,19, 23, 29)

2) The number of divisors of 120 including unity is _____. (C.D.S Exam)

Ans: 16. (ie, 1,2,3,4,5,6,8,10,12,15, 20,24,30,40,60,120)

3) The least number which is a perfect square and contains 180 as a factor is _____ (C.D.S Exam)

Ans: To get the answer, factorise 180.

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5$

2	180
2	90
3	45
3	15
	5

To get the perfect square which contains 180 as a factor it should be multiplied by 5.

Required number = $180 \times 5 = 900$ which is a perfect square.

✪ ✪ ✪

2. H. C. F AND L. C. M

H. C. F. - HIGHEST COMMON FACTOR

The HCF of two or more numbers is the highest among common factors. It is also referred to as Greatest Common Divisor (G. C. D)

Ex: Find the H. C. F of 12, 18 and 24 ?

Factors of 12 = 1, 2, 3, 4, 6, 12

" 18 = 1, 2, 3, 6, 9, 18

" 24 = 1, 2, 3, 4, 6, 8,12, 24.

Common factors = 1, 2, 3, 6

Highest Common Factor = **6**

Division Method

H. C. F of 12, 18, and 24 = 2 x 3 =6

2	12, 18, 24
3	6, 9, 12
	2, 3, 4

Problems :

1) Find out the H. C. F. of 1.5, 2 & 5 ?

Ans: $1.5 \times 10 = 15$

$2 \times 10 = 20$ H. C. F = 5

$5 \times 10 = 50$

5	15, 20, 50
	3 4 10

HFC of 15, 20 & 50 = 5

H.C.F. of 1.5, 2 and 5 = $\frac{5}{10}$ = **0.5**

2) If x is a multiple of y, then the highest common factor of 'x' and 'y' is ------- (U.S.S Scholarship Exam)

Ans : **y**

3) The length, breadth and height of a room are 8 m. 25 cm., 6m. 75 cm. and 4m. 50cm., respectively. Determine the longest tape which can measure the three dimensions of the room exactly? (Sainik School Admission Test)

Ans: 8m. 25 cm. = 825 cm.

6m. 75 cm. = 675 cm.

4m. 50 cm. = 450 cm.

3	825, 675, 450
5	275, 225, 150
5	55, 45, 30
	11, 9, 6

H.C.F of 825, 675 and 450 = $3 \times 5 \times 5 = 75$

∴ Measure of the tape = **75 cm.**

4) Find the largest number which exactly divides 29, 39 and 58 leaving remainders 2, 3 and 4, respectively ? (N.D.A Exam)

Ans: 29 - 2 = 27

39 - 3 = 36

58 - 4 = 54

(Before finding HCF, remainders should be deducted from the numbers)

3	27, 36, 54
3	9, 12, 18
	3, 4, 6

H. C. F of 27, 36, 54, = 3 × 3 = **9**

L. C. M - LOWEST COMMON MULTIPLE

L. C. M of two or more numbers is the lowest common multiple of the given numbers.

eg :- Find L. C. M of 6, 8, and 12 ?

Ans : Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48,.....

Multiples of 8 = 8, 16, 24, 32, 40, 48,.....

Multiples of 12 = 12, 24, 36 48, 60,.....

Common Multiples = 24, 48.

Lowest common Multiple = **24**

Division Method :

L.C.M of 6, 8, 12 = 2 × 2 × 3 × 1 × 2 × 1

= **24**

2	6, 8, 12
2	3, 4, 6
3	3, 2, 3
	1, 2, 1

Note:

Eventhough Division method for calculating HCF and LCM is found to be similar , there is one main difference . The lowest row of numbers where all the digits can not be divided by a prime number should not be taken to find HCF.

Ex: Find HCF and LCM of 12, 18 and 24 ?

H C F = 2×3=6

L C M = 2×3 × 2 × 1 × 3 × 2 = **72**

2	12, 18, 24
3	6, 9, 12
	2, 3, 4

2	12, 18, 24
3	6, 9, 12
2	2, 3, 4
	1, 3, 2

Problems :

1) The least number exactly divisible by 8, 12, 15 and 20 is ------- (Clerk's Grade Exam)

Ans : L. C. M of 8, 12, 15 and 20 is

= 2 × 2 × 3 × 5 × 2

= **120**

2	8, 12 , 15, 20
2	4, 6, 15, 10
3	2, 3, 15, 5
5	2, 1, 5, 5
	2, 1, 1, 1

2) Find out the L. C. M of 2, 3, 4, 6, 8, 12 and 16 ?

Ans: 2, 4, 8, are factors of 16

and 3,6 are factors of 12.

As such 2, 3, 4, 6, 8 need not be considered separately,

∴ L.C.M of 12 and 16 = 2 × 2 × 3 × 4

= **48**

2	12, 16
2	6, 8
	3, 4

3) The last number which when divided by 12, 15, 20 or 54, leaves in each case , a remainder of 4 is _____. [M. B. A. Entrance Exam]

Ans: L. C. M of 12, 15, 20 & 54

= 2 × 2 × 3 × 5 × 9

= 540

2	12, 15, 20, 54
2	6, 15, 10, 27
3	3, 15, 5, 27
5	1, 5, 5, 9
	1, 1, 1, 9

To get 4 as remainder we should add 4 to L.C.M. 540.

Answer is 540 + 4 = **544**.

4) Three measuring rods are 64 cm, 80 cm and 96 cm in length. The length of cloth (in metres) that can be measured exact number of times using any one of the above rods is_____.

[Inspectors of Income Tax. etc. Exam.]

Ans: The length of cloth will be lowest Common Multiple of rods.

L.C.M. of 64, 80 and 96 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 3$

= 960 cm.

= $\frac{960}{100}$

= **9.6 metres**

2	64, 80, 96
2	32, 40, 48
2	16, 20, 24
2	8, 10, 12
2	4, 5, 6
	2, 5, 3

PROPERTIES OF H. C. F AND L .C. M.

1) The H.C.F. of any two consecutive number is 1.

Ex: H.C.F. of 12 and 13 is 1.

2) The H.C.F. of two or more numbers can not be greater than the lowest of the numbers.

Ex: H. C. F. of 5, 10, and 15 is 5.

3) The L.C.M of two or more numbers can not be less than the highest of the numbers.

Ex: L.C.M. of 4 and 8 is 8.

4) The HCF of any two prime or Co-prime numbers is 1.

Ex: H.C.F. of 5 and 7 is 1.

5) The LCM of any two prime or Co-prime numbers equal their product.

Ex: L.C.M. of 5 and 7 = $5 \times 7 = 35$

6) The H.C.F. of two or more numbers is a factor of their LCM.

Ex: H C F of 8 and 12 is 4

L C M of 8 and 12 is 24

4 is a factor of 24.

7) H C F of fractions = $\frac{\text{HCF of numerators}}{\text{L C M of denominations}}$

eg : H C F of $\frac{2}{3}, \frac{4}{5}$ & $\frac{6}{15}$ = $\frac{\text{H.C. F of 2, 4, \& 6}}{\text{L.C.M of 3, 5, \&15}}$ = $\frac{2}{15}$

8) LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Ex: Find LCM of $\frac{2}{3}$, $\frac{1}{3}$ and $\frac{8}{3}$

(R. R. B examination)

Ans : $\frac{\text{LCM of 2, 1 and 8}}{\text{HCF of 3, 3 and 3}}$ = $\frac{8}{3}$ = $\mathbf{2\frac{2}{3}}$

9) The product H C F and L C M of two numbers equals their product

Ex: The H C F of two numbers is 16 while their L C M is 160. If one of the numbers is 32, What is the other number ?

[Hotel Management Diploma Course Entrance Exam]

Let other number be x

Ans: Product of two numbers = HCF $\times$ LCM

$32 \times x = 16 \times 160$

$x = \frac{16 \times 160}{32} = \mathbf{80}$

Problems :

1) If x and y are two integers whose LCM and HCF are 'l' and 'g' respectively, then (a) xg = ly, (b) xy = lg, (c) xl = gy, (d) None of these. (N D A Exam)

Ans: Product of numbers = LCM $\times$ HCF

$\therefore$ xy = **lg**

2) The highest common factor of two numbers is one, then their least common Multiple is _____ (CDS Exam)

Ans: The product of these two numbers.

Ex : LCM of 5 and 11 is $5 \times 11 =$ **55**

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3. FRACTIONS

A fraction is a part of whole number.

Ex: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{5}{3}$, $3\frac{1}{2}$ etc

$\frac{1}{4}$ 

Proper Fraction (സാധാരണ ഭിന്നം)

If the numerator of a fraction is less than its denominator, it is called a proper fraction and its value is always less than one.

Ex: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{3}{14}$ etc.

$\frac{1}{2}$ ← 1 ← Numerator (അംശം)
2 ← Denominator (ഛേദം)

Improper Fraction (വിഷമ ഭിന്നം)

If the numerator of a fraction is greater than or equal to its denominator, it is called an improper fraction.

Ex: $\frac{5}{3}$, $\frac{6}{6}$, $\frac{10}{5}$, etc.

Its value is always greater than or equal to 1.

Mixed Fraction (മിശ്ര ഭിന്നം)

A counting number + a proper fraction.

eg : $1\frac{1}{2}$, $3\frac{1}{3}$, $12\frac{3}{5}$, etc.

Equivalent Fraction

To get a fraction equivalent to a given fraction, multiply or divide the numerator and the denominator by the same number.

Ex: 1. $\frac{1 \times 2}{2 \times 2} = \frac{2 \times 3}{4 \times 3} = \frac{6 \div 2}{12 \div 2} = \frac{3}{6}$

2. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{400}{1200} = \frac{20}{60}$

Comparison of Fractions

1) If two fractions have the same denominator, the fraction with the larger numerator is greater than the other.

Ex: $\frac{1}{6} < \frac{2}{6} < \frac{5}{6} > \frac{3}{6} < \frac{9}{6}$

'>' represents "is greater than"
'<' represents "is less than"
Smaller side → < ← Greater side

2) If two fractions have equal numerators, the fraction with the smaller denominator is greater than the other.

Ex: $\frac{5}{3} < \frac{5}{2} > \frac{5}{4} < \frac{5}{1}$

3) If the numerators and denominators are different, find LCM of denominators.

Ex: Which of the following fractions is the largest ?

$\frac{3}{5}$ & $\frac{4}{6}$

Ans: For comparison of fractions, denominators should be same. Therefore, find LCM of denominators.

LCM of 5 & 6 = $5 \times 6 = 30$

$\frac{3 \times 6}{5 \times 6}, \frac{4 \times 5}{6 \times 5} = \frac{18}{30} < \frac{20}{30}$

Largest fraction is $\frac{20}{30}$, ie $\frac{\mathbf{4}}{\mathbf{6}}$

Shortcut Method:-

If $\frac{a}{b}$ & $\frac{c}{d}$ are two fractions, $\frac{a}{b} \times \frac{c}{d}$

and If $a \times d > b \times c$, then $\frac{a}{b} > \frac{c}{d}$

If $a \times d < b \times c$ then $\frac{a}{b} < \frac{c}{d}$

$\frac{3}{5}$ & $\frac{4}{6}$, 3×6 and 5×4, $18 < 20$, $\frac{3}{5}$, $< \frac{4}{6}$

Largest fraction is $\mathbf{\frac{4}{6}}$

Conversion of improper Fraction into Mixed fraction.

eg :- 1) $\frac{7}{3} = 2\frac{1}{3}$

$3\overline{)7}$ quotient 2, remainder 1

2) $\frac{13}{5} = 2\frac{3}{5}$

Conversion of Mixed fraction into Improper fraction.

Ex:- 1) $2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{6+1}{3} = \frac{7}{3}$

To get the numerator, multiply the counting number by denominator and add the numerator. Denominator will be same.

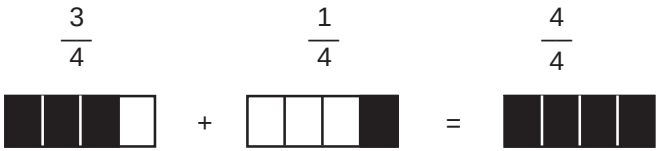
2) $5\frac{2}{3} = \frac{15+2}{3} = \frac{17}{3}$ (3) $7\frac{1}{7} = \frac{50}{7}$

(No change in denominator)

Addition and subtraction of Fractions

[i] Fraction with same denominators :

Ex: **1]** $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$



Add numerators only, denominator will be same.

2] $3\frac{1}{2} + 2\frac{1}{2} = \frac{7}{2} + \frac{5}{2} = \frac{12}{2} = 6$

[Convert mixed fractions into improper fractions and add the fractions]

3] $1 + \frac{3}{7} = \frac{7}{7} + \frac{3}{7} = \frac{10}{7} = 1\frac{3}{7}$

[To add '1' to a fraction , convert the number '1' into fraction so as to equal the denominator of the other fraction. If final answer is an improper fraction convert it into a mixed fraction]

4] $1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8} = \frac{5}{8}$

5] $7\frac{3}{5} + 3\frac{1}{5} - 4\frac{2}{5} = ?$

Ans : *Method (1)* Addition and subtraction after converting into improper fractions

$= \frac{38}{5} + \frac{16}{5} - \frac{22}{5} = \frac{38+16-22}{5} = \frac{32}{5} = 6\frac{2}{5}$

Method (2) Whole numbers and fractions separately doing :

$= (7 + 3 - 4) + \left(\frac{3}{5} + \frac{1}{5} - \frac{2}{5}\right)$

$= 6 + \frac{3+1-2}{5} = 6 + \frac{2}{5} = 6\frac{2}{5}$

[II] Fractions with different denominators:

In order to find the sum or difference of two or more fractions, first we find the LCM of their denominators. Then we multiply each fraction by their LCM. The LCM will be the denominator of the resulting fraction.

Ex: **1]** $\frac{3}{4} + \frac{2}{5} - \frac{7}{10} = ?$

$$\begin{array}{r|l} 2 & 4,\ 5,\ 10 \\ 5 & 2,\ 5,\ 5 \\ \hline & 2,\ 1,\ 1 \end{array}$$

Ans: LCM of 4, 5 and 10 $= 2 \times 5 \times 2 = 20$

Multiply the fractions with L C M.

$$= \frac{\frac{3}{4} \times 20 + \frac{2}{5} \times 20 - \frac{7}{10} \times 20}{20}$$

$$= \frac{(3 \times 5) + (2 \times 4) - (7 \times 2)}{20}$$

$$= \frac{15 + 8 - 14}{20} = \frac{23 - 14}{20} = \mathbf{\frac{9}{20}}$$

Another Method :

$$= \frac{3 \times 5}{4 \times 5} + \frac{2 \times 4}{5 \times 4} - \frac{7 \times 2}{10 \times 2}$$

$$= \frac{15}{20} + \frac{8}{20} - \frac{14}{20} = \mathbf{\frac{9}{20}}$$

2] $5\frac{1}{3} + 3\frac{1}{2} - 4\frac{1}{4} =$ ____ (PSC Divisional Accountant Test)

Ans : $(5 + 3 - 4) + (\frac{1}{3} + \frac{1}{2} - \frac{1}{4})$ (LCM of 3,2, & 4 is 12)

$$= 4 + \left[\frac{1 \times 4}{12} + \frac{1 \times 6}{12} - \frac{1 \times 3}{12}\right]$$

$$= 4 + \frac{4 + 6 - 3}{12}$$

$$= 4 + \frac{7}{12} = \mathbf{4\frac{7}{12}}$$

3] $4\frac{4}{2} + 3\frac{3}{5} + 2\frac{6}{5} = ?$ (Bank P. O Exam.)

Ans : $\frac{12}{2} + \frac{18}{5} + \frac{16}{5} = 6 + \frac{18 + 16}{5} = 6 + \frac{34}{5}$

$$= 6 + 6\frac{4}{5} = \mathbf{12\frac{4}{5}}$$

4] $6\frac{1}{2} + 3\frac{1}{4} - 2\frac{3}{4} = ?$ (Bank P. O Exam.)

Ans: $(6 + 3 - 2) + \left(\frac{1}{2} + \frac{1}{4} - \frac{3}{4}\right)$

$$= 7 + \frac{2 + 1 - 3}{4} = 7 + \frac{0}{4} = 7 + 0 = \mathbf{7}$$

Multiplication of Fractions

Product of Fractions $= \frac{\text{Product of numerators}}{\text{Product of denominators}}$

Ex : 1) $\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \mathbf{\frac{3}{10}}$

2) $\frac{2}{3} \times 1\frac{1}{2} = \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = \mathbf{1}$

3) $3\frac{1}{3} \times 7 = \frac{10}{3} \times \frac{7}{1} = \frac{70}{3} = \mathbf{23\frac{1}{3}}$

Reciprocals (വ്യുൽക്രമം)

eg : 1) Reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$

2) Reciprocal of $\frac{6}{4}$ is $\frac{4}{6}$

(Numerators and denominators are interchanged)

3) Reciprocal of 5 is $\frac{1}{5}$

(5 means $\frac{5}{1}$, any number divided by 1 is same number)

4) Reciprocal of $3\frac{1}{2}$ = ...

First convert $3\frac{1}{2}$ in to improper fraction.

$3\frac{1}{2} = \frac{7}{2}$, Reciprocal is $\frac{2}{7}$

Division of Fractions

Fraction ÷ Divisor = Fraction x Reciprocal of Divisor

1) $\frac{2}{3} \div \frac{4}{6} = \frac{2}{3} \times \frac{6}{4} = \frac{12}{12} = \mathbf{1}$

2) $3\frac{1}{4} \div 5 = \frac{13}{4} \div \frac{5}{1} = \frac{13}{4} \times \frac{1}{5} = \mathbf{\frac{13}{20}}$

3) $7 \div \frac{5}{6} = 7 \times \frac{6}{5} = \frac{42}{5} = \mathbf{8\frac{2}{5}}$

(First number will not change)

4) Reduce to decimal : $\frac{3}{8} \times 1\frac{1}{3}$

Ans : $\frac{3}{8} \times \frac{4}{3} = \frac{12}{24} = \frac{1}{2} = \mathbf{0.5}$

5). $\frac{19}{3} \div \frac{38}{6} \times \frac{4}{3} = ?$ (SBI P.O Exam)

$\frac{19}{3} \times \frac{6}{38} \times \frac{4}{3} = \frac{2 \times 2}{3} = \frac{4}{3} = \mathbf{1\frac{1}{3}}$

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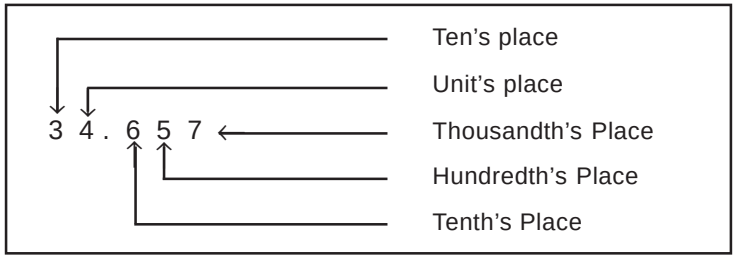
4. DECIMALS

Ex. A decimal is a fraction which has 10 or any power of 10 as its denominator, and is expressed in the decimal system of notation.

Ex : 1.3, 0.5, 83.075 etc.

0.5 reads as “point Five”

83. 075 reads as “Eighty three point Zero, Seven, Five”



a) $34.657 = 34 + \frac{6}{10} + \frac{5}{100} + \frac{7}{1000}$

b) $4 + \frac{1}{10} + \frac{5}{1000} = 4.105$

c) $\frac{809}{100} = 8.09$

$0.1 = \frac{1}{10}$

$0.01 = \frac{1}{100}$

$0.001 = \frac{1}{1000}$

Addition and subtraction of Decimals

Addition and Subtraction of decimal numbers are carried out in exactly the same way as with whole numbers, provided the decimal points are kept in vertical line.

Ex: 1] 8.24 + 3.36 + 5.4 = 17.00

```
 8.24 +
 3.36
 5.40
-------
17.00
-------
```

2] 0.078 + 1.2 = 1.278

$$\begin{array}{r} 0.078 \\ +\ 1.200 \\ \hline 1.278 \end{array}$$

3] 14.83 – 1.75 = 13.08

$$\begin{array}{r} 14.83 \\ -\ 1.75 \\ \hline 13.08 \end{array}$$

4] 10.3 – 8.72 = 1.58

$$\begin{array}{r} 10.30 \\ -\ 8.72 \\ \hline 1.58 \end{array}$$

5] 2 – 1.005 = 0.995

$$\begin{array}{r} 2.000 \\ -\ 1.005 \\ \hline 0.995 \end{array}$$

Problems

1) 25.3 + 0.32 + 2.123 is equal to ____ . {Clerks Grade Exam)

Ans:

$$\begin{array}{r} 25.300 \\ +\ 0.320 \\ \quad 2.123 \\ \hline 27.743 \end{array}$$

2) 0. 001 - 0.0001 = ... [Bank Clerical Exam]

Ans:

$$\begin{array}{r} 0.0010 \\ -\ 0.0001 \\ \hline 0.0009 \end{array}$$

3) What is the fraction does 0.999 represent ?

[R.R.B. Exam., (Bombay)]

Ans : $0.999 = \frac{999}{1000}$

(Number of decimal places will be equal to number of zeros after '1' in denominator)

4) Write in ascending and descending orders.

2, 0.999, 1, 1.5, 2.099, 2.1, 9.999, 10.01

Ascending order→ 0.999, 1, 1.5, 2, 2.099, 2.1, 9.999, 10.01

Descending order → 10 01, 9.999, 2.1, 2.099, 2, 1.5, 1, 0.999.

Multiplication and Division of Decimals

Multiplication :

Ex: 1] 5.85 × 7 = 40.95

2] 5.85 × 0.7 = 4.095

3] 0.001 × 0.01 = 0.00001

$$\begin{array}{r} 5.85 \times \\ 7 \\ \hline 40.95 \end{array}$$

The number of decimal places in product will be equal to the sum of the number of decimal places in the multiplier and the multiplicant.

Number of decimal places in 0.001 = 3

Number of decimal places in 0.01 = 2

Sum = 3 + 2 = 5

∴ Number of decimal places in product = 5

ie 0.001 × 0.01 = 0.00001

4] 0.01 x 0.01 x 0.001 = ?

(U.S.S., Scholarship Exam)

Ans: 0.0000001

Division :

[i] Division by a whole number :

Ex: 1) 16.512 ÷ 3 = 5.504

2) 165.12 ÷ 3 = 55.04

3) 1.6512 ÷ 3 = 0.5504

4) 510 ÷ 25 = 20.4

$$\begin{array}{r|l} & 5.504 \\ 3 & 16.512 \\ & 15 \\ \hline & 15 \\ & 15 \\ \hline & 12 \\ & 12 \\ \hline & 00 \end{array} \qquad \begin{array}{r|l} & 20.4 \\ 25 & 510 \\ & 50 \\ \hline & 100 \\ & 100 \\ \hline & 000 \end{array}$$

[ii] Division by a Decimal

To divide one decimal by another one, convert the denominator into a whole number. To get a whole number multiply both numerator and denominator by 10 or 100, as the case may be.

Ex: 1) $\frac{3.45}{0.3} = \frac{3.45 \times 10}{0.3 \times 10} = \frac{34.5}{3} =$ **11.5**

$$
\begin{array}{r|l}
 & 11.5 \\
\hline
3 & 34.5 \\
 & 3 \\
\hline
 & 4 \\
 & 3 \\
\hline
 & 15 \\
 & 15 \\
\hline
 & 00
\end{array}
$$

2) 808 ÷ 8 ÷ 0.4 = ?

(Bank P.O. Exam,)

Ans : 808 ÷ 8 = 101

$101 \div 0.4 = \frac{101 \times 10}{0.4 \times 10} = \frac{1010}{4} =$ **252.5**

3) 5.626 - 1.05 = ?

Ans: $\frac{5.626 \times 100}{1.05 \times 100} = \frac{562.6}{105} =$ **5.3581**

The answer 5.3581 correct to 3 decimal places is 5.358

correct to 2 decimal places is 5.36

To get a number multiplied by 10, 100, etc.,

shift the decimal point to right as shown below.

To multiply by 10, shift decimal point one place to the right.

To multiply by 100, shift decimal Point two places to the right and so on.

15.16 × 10	= 151.6
15.16 × 100	= 1516
15.16 × 1000	= 15160
15.16 × 10000	= 151600

For division by 10, 100, 1000, etc. shift decimal point to left as shown below.

15.16 ÷ 10	= 1.516
15.16 ÷ 100	= 0.1516
15.16 ÷ 1000	= 0.01516
15.16 ÷ 10000	= 0.001516

$\frac{1}{2}$	= 0.50
$\frac{1}{3}$	= 0.33
$\frac{1}{4}$	= 0.25
$\frac{1}{5}$	= 0.20

Problems :

1) If $\frac{3485}{20.5} = 170$, then $\frac{34.85}{2.05} =$ ..... (Bank P.O. Exam)

Ans: **17**

2) 17.808 + 8 + 0.4 = _____ (P.S.C Divisional Accountant Test)

Ans:

$$
\begin{array}{r}
17.808 + \\
8.000 \\
0.400 \\
\hline
26.208
\end{array}
$$

3) .600 ÷ .15 - .05 × 8 × 5 + .500 × 4 = _____

(L.I.C. Asst. Grade II Exam.)

Ans: $= \frac{0.600 \times 100}{0.15 \times 100} - (.05 \times 8 \times 5) + .500 \times 4$

$= \frac{60}{15} - 2 + 2 = 4 - 2 + 2 = 2 + 2 =$ **4**

4) Three-fourth of a tank is full of water. If 5 litres are added to it then four-fifth of tank become full. What is the capacity of the tank ? (P S C Test)

Ans: $(\frac{4}{5} - \frac{3}{4})$ th of tank capacity = 5 litres

$\frac{4 \times 4 - 3 \times 5}{20} = \frac{16-15}{20} = \frac{1}{20}$ th of tank capacity = 5 litres

Tank capacity = $\frac{5 \times 20}{1} =$ **100 litres**

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5 ALGEBRAIC EXPRESSIONS

A. Like and unlike Terms.

B. Operations with Positive and Negative Numbers.

C. Linear Equations.

D. Simultaneous Equations.

E. Quadratic Equations.

F. Polynomials.

G. Identities

H. Laws of Exponents.

I. Application of Algebraic Equations to Practical problems.

Algebraic Expressions

A number or a combination of numbers (including variables), using the signs of fundamental operation(s) is called an algebraic expression.

The word, 'Algebra ' is derived from the title of the book 'Algebar W' al almugabalah' writtan about A.D.825 by an Arab mathematician. Mohammed ibn Al Khowarizmi of Baghdad. Great Indian Mathematicians Aryabhatta (born in A. D. 476), Brahmagupta (born in A. D. 598), Mahavira (around A. D. 850), Sridhara (around A.D.1025), and Bhaskara II (born in A. D. 1114) contributed a lot to the study of algebra.

A. LIKE AND UNLIKE TERMS

Terms: A term contains variable(s) and constant with the operations of multiplication or division.

Ex:- 4a, 3xy, $\frac{2x}{y,}$ etc.

Variables: A quantity which takes on various numerical values in different situations is called a variable. It has no fixed value. Its value may vary in different circumstances. Usually variables are denoted by small letters of English Albabets like x, y, z, etc.

Ex:- (1) Radius of different circles.

(2) Distance between two vehicles moving in a road.

4**×**a can be written as 4a. 4 is constant. and a is variable

1x can be written as x. 1 is constant. and x is variable.

Like Terms : Terms containining same variables are called like terms.

Ex:- 3xy, 5xy, 4yx, etc.

Unlike Terms : Terms containing different variables are called unlike terms.

Ex:- 3x, 4y, 5xy, 7p, etc.

Addition and Subtraction of like terms

Ex:- 1) $8p + 7p + P = (8 + 7 + 1)P =$ **16P**

2) $9x + 7x - x = (9 + 7 - 1)x =$ **15x**

3) Subtract 8x from the Sum of 3x and 18x
Sum of 3x and 18x = 3x + 18x = 21x
21x - 8x = (21- 8)x = **13x**

4) $3x^2 + 2x^2 + x^2 = (3 + 2 + 1)x^2 =$ **$6x^2$**

Addition and Subtraction of unlike terms

Ex:- 1) Find the sum of 6x, 3y, x, 3z and y.

Ans: 6x + 1x + 3y + 1y + 3z = **7x + 4y + 3z**

2) 5x - 4y + 2x - 2y = ?

Ans: 5x + 2x - 4y - 2y = **7x - 6y**

Multiplication and Division of terms

Ex:- 1) $5p \times 12q = (5 \times 12) \times (p \times q) = 60 \times pq =$ **60pq**

2) $3a \times b \times c \times 5 = 3 \times 5 \times a \times b \times c =$ **15 abc**

3) $a \times a \times a =$ **a^3**

4) $5x \times 2x \times 3 = 5 \times 2 \times 3 \times x \times x =$ **$30x^2$**

5) $40a \div 8a = \frac{40a}{8a} =$ **5**

6) $4a^3 \div 2a = \frac{4a^3}{2a} = \frac{4 \times a \times a \times a}{2 \times a} =$ **$2a^2$**

7) $\frac{4x}{8x^2} = \frac{4 \times X}{8 \times X \times X} =$ **$\frac{1}{2x}$**

8) $\frac{3x^2}{15xy} \times \frac{5y}{3xy} = \frac{3 \times 5 \times X \times X \times y}{15 \times 3 \times X \times X \times y \times y} =$ **$\frac{1}{3y}$**

9) $16 \times 16 \times 16 - 8 \times 8 \times ? =$ **0**

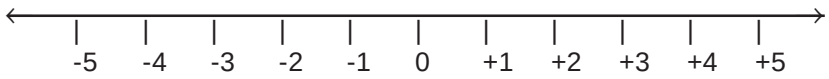
Ans. $16 \times 16 \times 16 - 8 \times 8 \times X = 0$

$8 \times 8 \times X = 16 \times 16 \times 16$

$X = 2 \times 2 \times 16 =$ **64**

B. OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS

Number Line: (ie line with numbers)



Value of numbers increases towards right.

Value of numbers decreases towards left.

ie, +3 is greater than +2, +1, 0, -1, etc.

-5 is less than -4, -3, -2, -1, 0, +1 etc.

-1 is greater than -1000

Ex : Write in ascending order : +5, -50, 0, +2, -999

Ascending order : -999, -50, 0, +2, +5

Addition:

Ex:- 1) $+5 + +7 = +12$

2) $-20 + -20 = -40$

3) $+20 + -15 = 20 - 15 = +5$

To add the numbers of different signs, subtract the smaller one from the greater one and put the sign of the greater one.

Subtraction:

For Subtraction, add the additive inverse of the 2nd term.

Ex:- 1) $+18 - +8 = +18 + -8 = 10$

2) $+18 - -8 = +18 + +8 = +26$

3) $-18 - +8 = -18 + -8 = -26$

4) $-18 - -8 = -18 + +8 = -10$

$x - y = x + (-y)$

$-(-x) = +x$

Number	Additive inverse
+3	-3
-5	+5
$+\frac{1}{2}$	$\frac{-1}{2}$

Multiplication and Division

The sign of product or quotient will be positive if the numbers have same signs.

The signs of product or quotient will be negative if the numbers have different signs.

Ex:- 1) $+8 \times +4 = +32$

$-8 \times -4 = +32$

$+8 \times -4 = -32$

$-8 \times +4 = -32$

(2) Divide the sum of -8 and +6 by their difference.

Ans: Sum $= -8 + +6 = -2$

Difference $= -8 - +6 = -8 + -6 = -14$

$$\frac{\text{Sum}}{\text{Difference}} = \frac{-2}{-14} = \frac{1}{7}$$

(For positive numbers, no need of writing "+" sign. ie +5 = 5)

$+ \times + = +$	$+ \div + = +$
$- \times - = +$	$- \div - = +$
$+ \times - = -$	$+ \div - = -$
$- \times + = -$	$- \div + = -$

C. LINEAR EQUATIONS

Equation:

An equation is a Statement where two algebraic expressions are equal.

Ex:- 1) $x + 3 = 8$, 2) $2x + 3 = 7$,

3) $4y + x = x + 4y$

Linear Equation:

An equation containing one or more unknown quantities is said to be linear equation (or first degree equation) when the unknown occures only in the first power.

Ex:- 1) $ax + b = 0$, 2) $ax + by = c$,

3) $2x + 3 = 7$

Numeral Statement:

A statement which does not contain variable is called numeral statement.

Ex:- $5 + 4 < 16$, $8 + 2 = 10$, etc.

Open Sentence:

A sentence which contain variable or variables is called an open sentence.

Ex:- $x + 3 > 5$, $x - 2 < 6$, $2x + y = 10$

Find the value of x in the following equations?

1) $x + 5 = 8$

$x = 8 - 5 = \mathbf{3}$

2) $x - 5 = 8$

$x = 8 + 5 = \mathbf{13}$

3) $5x = 10$

$x = \frac{10}{5} = \mathbf{2}$

4) $\frac{x}{5} = 8$

$x = 8 \times 5 = \mathbf{40}$

5) $\frac{6x + 3}{4} = 2$

$6x + 3 = 2 \times 4 = 8$

$6x = 8 - 3 = 5$

$x = \frac{\mathbf{5}}{\mathbf{6}}$

6) $\frac{6x}{4} - 3 = 2$

$\frac{6x}{4} = 2 + 3 = 5$

$6x = 5 \times 4 = 20$

$x = \frac{20}{6} = \frac{10}{3} = \mathbf{3\frac{1}{3}}$

(To find the value of X, transpose the other terms to right. By transposing a term means change its sign and taking it to other side of the equation.)

Cross Multiplication:

Ex:- 1) $\frac{x}{3} = \frac{2}{6}$

Product of numerator of the first term and denominator of the second term will be equal to product of denominator of the first term and numerator of the second term.

$\frac{x}{3} \times \frac{2}{6}$

$x \times 6 = 3 \times 2$

$6x = 6$

$x = \frac{6}{6} = \mathbf{1}$

2) $\frac{1}{x} = 5$

$\frac{1}{x} = \frac{5}{1}$ (Hint: $5 = \frac{5}{1}$, any number divided by 1 is same)

By cross multiplication, $x \times 5 = 1 \times 1$

$$x = \frac{\mathbf{1}}{\mathbf{5}}$$

Problems

1) $\frac{x - 10}{3} = \frac{4x + 9}{4}$ Find the value of x.

(I.T.I. Apprentice Test)

Ans: $\frac{x - 10}{3} = \frac{4x + 9}{4}$

By cross multiplication, $3(4x + 9) = 4 (x - 10)$

$12x + 27 = 4x - 40$

(Transfer the numbers with variable part into left side and numerals into right side by changing signs.)

$\therefore 12x - 4x = -40 - 27$

$8x = -67$

$x = \frac{-67}{8} = -8.375$

2) $\frac{4}{5}$ of a certain number is 64. Half of that number is ____.

(Bank P.O. exam.)

Ans: Let the number be X.

$\therefore \frac{4}{5}$ of $X = 64$

$\frac{4}{5} \times X = 64$

$X = 64 \times \frac{5}{4} = 80$

Half of the number $= \frac{1}{2} \times 80 = \mathbf{40}$

3) If one-third of one-fourth of a number is 12, then find the number ?

(Bank P.O. Exam.)

Ans: Let the number be x

$\frac{1}{3}$ of $\frac{1}{4}$ of $X = 12$

$\frac{1}{3} \times \frac{1}{4} \times X = 12$

$\frac{1}{12} X = 12$

$X = 12 \times 12 = \mathbf{144}$

4) Find the value of a property if a man possessing $\frac{3}{5}$ of it and sells $\frac{2}{5}$ of his share for Rs.12,000 ?

(R.R.B. Exam.)

Ans: Suppose value of the property = Rs. x

His share $= \frac{3}{5} x$

Sales $= \frac{2}{5}$ of $\frac{3}{5} x$

$\frac{2}{5} \times \frac{3}{5} x =$ Rs. 12,000

$\frac{6x}{25} =$ Rs. 12,000

$X =$ Rs.12,000 $\times \frac{25}{6}$

$=$ **Rs. 50,000**

D. SIMULTANEOUS EQUATIONS

When two equations with two unknowns are satisfied by the same pair of values for the two unknowns, they are said to be simultaneous equations.

Ex:- 1) The sum of two numbers is 7 and their difference is 3. Find the numbers?

Ans: Let the numbers be x and y.

$$x + y = 7 \quad \text{-- (1)}$$

$$x - y = 3 \quad \text{-- (2)}$$

(1) + (2) $\quad 2x + 0 = 10$

$$x = \frac{10}{2} = 5$$

Substituting the value of x in equation No. (1)

$$x + y = 7$$

$$5 + y = 7$$

$$y = 7 - 5 = 2$$

The numbers are 5 and 2.

E. QUADRATIC EQUATION

The equation in which the variable is at second order is called quadratic equation.

(i) Pure Quadratic Equation:

Ex:- $x^2 = k$

(ii) Quadratic Equation:

$ax^2 + bx + c = 0$, Where $a \neq 0$

Solution of a Quadratic Equation by standard formula:

$$\text{Roots of a quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If roots are α and β,

$$\text{Sum of solutions } (\alpha + \beta) = \frac{\text{Negative of Co-efficient of } x}{\text{Co- efficient of } x^2} = \frac{-b}{a}$$

$$= \left[\alpha + \beta = \frac{-b}{a}\right]$$

$$\text{Product of solutions } (\alpha\,\beta) = \frac{\text{Constant term}}{\text{Co- efficient of } x^2} = \frac{c}{a}$$

Problems:

1) Solve the equation $7x^2 = 8 - 10x$ $\quad a = 7,\ b = 10,\ c = -8$

Ans: $7x^2 + 10x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2 - 4 \times 7 \times -8}}{2 \times 7}$$

$$= \frac{-10 \pm \sqrt{100 + 224}}{14}$$

$$= \frac{-10 \pm \sqrt{324}}{14}$$

$$= \frac{-10 + 18}{14} \text{ and } \frac{-10 - 18}{14}$$

$$= \frac{8}{14} \text{ and } \frac{-28}{14}$$

$$= \frac{4}{7} \text{ and } -2$$

Solution Set $= \left\{\frac{4}{7}, -2\right\}$

2) The sum of two numbers is 60 and their product is 360. Then the sum of their reciprocals is _____

(C. D. S. Exam.)

Ans: Let α and β are two numbers

$\alpha + \beta = 60$

$\alpha\beta = 360$

Then, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{60}{360} = \mathbf{\frac{1}{6}}$

F. POLYNOMIALS

'Poly' means many. A polynomial has more than two terms.

Two or more signs (+ and -) separate an expression into several parts. Each part along with its sign is called a term of the expression.

Monomials : It has one term.

Ex:- 3x, 5ab, $\frac{-6x}{2}$, xyz, etc.

Binomials : It has two terms.

Ex:- 3x +6y, $3x^2$ +5ab, etc.

Trinomials : It has three terms.

Ex:- 3x + 5ab + c, x + y - z, etc.

The terms having same variable(s) and of the same degree are like terms.

Ex:- Like terms of 1st degree variables $\rightarrow$ x, 2x, 4x, etc.

Like terms of 2nd degree variables $\rightarrow$ x^2, $2x^2$, $4x^2$, etc.

Like terms of 3rd degree variables $\rightarrow$ y^3, $2y^3$, $5y^3$, etc.

Operations with Polynomials

Addition and Subtraction

In order to add or subtract the polynomials, arrange like terms in one column and add or subtract the co-efficients of each column separately.

Ex:- 1) Find the sum of 5x + 6y - 7 and 3y - x + 5

Ans:

$$\begin{array}{r} 5x + 6y - 7 + \\ -1x + 3y + 5 \\ \hline 4x + 9y - 2 \end{array}$$

2) Subtract $5x^2 - 8x + 20$ from $7x^2 + 16x + 4$

Ans: $7x^2 + 16x + 4 -$

$5x^2 - 8x + 20$

Add the additive inverse of 2nd polynomial.

$$\begin{array}{r} 7x^2 + 16x + 4 + \\ -5x^2 + 8x - 20 \\ \hline 2x^2 + 24x - 16 \end{array}$$

Multiplication

For multiplying a monomial with a binomial, multiply each term of the binomial by the monomial and add the products.

Ex: $-2x^2(5x^2 - 9x - 6)$ (multiply each term in the bracket by $-2x^2$)

$= (-2x^2 \times 5x^2) + (-2x^2 \times -9x) + (-2x^2 \times -6)$

$= \mathbf{-10x^4 + 18x^3 + 12x^2}$

For multiplying any two binomials, multiply each term of one binomial by each term of the other and add the products.

Ex:- Multiply (m-2) and (m+2)

Ans: Multiply the 2nd polynomial by each term of the 1st polynomial.

$(m-2)(m+2) = m(m+2) + -2(m+2)$

$= m^2 + 2m + -2m - 4$

$= m^2 + 0 - 4 = \mathbf{m^2 - 4}$

Division

For dividing a polynomial by a monomial, divide each term of the polynomial by the monomial.

Ex:- Divide $12xy - 16x^2 + 4x$ by $4x$

Ans: $\frac{12xy - 16x^2 + 4x}{4x} = \frac{12xy}{4x} - \frac{16x^2}{4x} + \frac{4x}{4x} =$ **3y - 4x +1**

G. IDENTITIES

If two phrases are connected by any of the symbols =, <, >, $\neq$, $\geq$, $\leq$, then it is called a sentence.

Ex:- $16 + 3 = 19$, $x + 2 \leq 10$, $3a + b = 0$

Open Sentences

If a sentence contains at least one variable it is called an open sentence.

Ex:- $x + 5 = 7$, $2x + y = 5$, etc.

Truth Set

The set of values from the domain of the variable that makes an open sentence true is called the truth set of that sentence.

Always True Sentences

In an open sentence, if the domain of the variable and the truth set are identical it is an always true sentence.

Ex:- (1) $4(x + 1) = 4x + 4$

(2) $7 + 3 > 5$

Identities

An always true sentence which contains '=' is termed as an identity.

Ex:- (1) $4(x+1) = 4x + 4$

(2) $2(a+b) = 2a + 2b$

IDENTITIES

$a(b+c)$	=	$ab+ac$
$k(a+b+c)$	=	$ka+kb+kc$
$(a+b)^2$	=	$a^2+2ab+b^2$
$(a-b)^2$	=	$a^2-2ab+b^2$
a^2+b^2	=	$(a+b)^2-2ab$
a^2-b^2	=	$(a+b)(a-b)$
$(a+\frac{1}{a})^2$	=	$a^2+\frac{1}{a^2}+2$
$(a-\frac{1}{a})^2$	=	$a^2+\frac{1}{a^2}-2$
$(a+b+c)^2$	=	$a^2+b^2+c^2+2ab+2bc+2ac$
$(a+b)(c+d)$	=	$ac+ad+bc+bd$
$(x+a)(x+b)$	=	$x^2+(a+b)x+ab$
$(a+b)^3$	=	$a^3+b^3+3ab(a+b)$
$(a-b)^3$	=	$a^3-b^3-3ab(a-b)$
a^3+b^3	=	$(a+b)^3-3ab(a+b)$
''	=	$(a+b)(a^2-ab+b^2)$
a^3-b^3	=	$(a-b)^3+3ab(a-b)$
''	=	$(a-b)(a^2+ab+b^2)$

PASCAL TRIANGLE

Co-efficient of the terms in the Expansion :

$[a+b]^0$ 1

$[a+b]^1$ 1 1

$[a+b]^2$ 1 2 1

$[a+b]^3$ 1 3 3 1

$[a+b]^4$ 1 4 6 4 1

$[a+b]^5$ 1 5 10 10 5 1

IDENTITIES: EXPANSION

$[a+b]^0 = 1$

$[a+b]^1 = a + b$

$[a+b]^2 = a^2 + 2ab + b^2$

$[a+b]^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$[a+b]^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$[a+b]^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Application of Identities

In a competitive examination, the most important element is that of time. Thus, utilisation of shortcut methods is very essential for solving the problems in time.

Problems :

1) The sum of two numbers is 100 and their difference is 37.
The difference of their squares is _____
(Clerks Grade Exam.)

Ans. Let the numbers are a and b.
then, $a+b = 100$, $a - b = 37$
Difference of their squares $= a^2 - b^2 = (a+b)(a-b)$
$= 100 \times 37 =$ **3,700**

2) $(1000)^2 - (999)^2$ is equal to _____ (Clerks Grade Exam.)

Ans. $a^2 - b^2 = (a+b)(a-b)$
$= (1000 + 999)(1000 - 999)$
$= 1999 \times 1 =$ **1999**

3) $[99]^2$ =?

Ans. $[99]^2 = [100 - 1]^2$
$[a - b]^2 = a^2 - 2ab + b^2$
$(100 - 1)^2 = 100^2 - 2 \times 100 \times 1 + 1^2$
$= 10000 - 200+1$
$= 9800 + 1 =$ **9801**

4) $12 \times 12 + 2 \times 12 \times 8 + 8 \times 8 = ?$

Ans. $a^2 + 2ab + b^2 = (a+b)^2$
$12^2 + 2 \times 12 \times 8 + 8^2 = (12+8)^2 = [20]^2 =$ **400**

5) 107×103 =?

Ans. $[x+a][x+b] = x^2 + (a+b)x + ab$
$[100+7][100+3] = 100^2 + (7+3)\,100 + 7 \times 3$
$= 10000 + 1000 + 21$
$=$ **11021**

6) If $x + \frac{25}{x} = 10$, then x = ?

Ans. $\frac{x}{1} + \frac{25}{x} = 10$ multiplying throughout by x we get

$x^2 + 25 = 10x$

$\therefore\ x^2 - 10x + 25 = 0$

$[x - 5]^2 = 0$

$x - 5 = \sqrt{0} = 0$

$x = 0 + 5 = \mathbf{5}$

7) $\frac{69842 \times 69842 - 30158 \times 30158}{69842 - 30158} = ?$ (Clerk Grade Exam.)

Ans. $\frac{a^2 - b^2}{a - b} = \frac{(a+b)(a-b)}{(a-b)} = a + b$

$= 69842 + 30158$

$= \mathbf{100000}$

8) Simplify $(.538)^2 - (.462)^2$ = _____.

(P. S. C. Divisional Accountant Test)

Ans. $a^2 - b^2 = [.538 + .462][.538 - .462]$

$= 1 \times .076 = \mathbf{0.076}$

9) $\frac{7.5 \times 7.5 \times 7.5 - 2.5 \times 2.5 \times 2.5}{7.5 \times 7.5 + 2.5 \times 7.5 + 2.5 \times 2.5} = ?$

(Bank P.O. Exam)

Ans. $\frac{a^3 - b^3}{a^2+ab+b^2} = \frac{(a-b)(a^2+ab+b^2)}{[a^2+ab+b^2]} = (a - b)$ a = 7.5, b = 2.5

$= 7.5 - 2.5 = \mathbf{5}$

10) $\frac{0.01 \times 0.01 \times 0.01 + 0.1 \times 0.1 \times 0.1}{0.03 \times 0.03 \times 0.03 + 0.3 \times 0.3 \times 0.3} = ?$ (Bank P.O. Exam)

Ans. Let $0.01 = a$, $\therefore\ 0.03 = 3a$

Let $0.1 = b$, $\therefore\ 0.3 = 3b$

$\frac{a^3+b^3}{(3a)^3+(3b)^3} = \frac{a^3+b^3}{27a^3+27b^3} = \frac{(a^3+b^3)}{27(a^3+b^3)} = \mathbf{\frac{1}{27}}$

H. LAWS OF EXPONENTS (OR LAWS OF INDICES)

$x+x+x+x = 4x$ x is variable and 4 is co-efficient of x.

$4 \times x = 4.x = 4x$

$x \times x \times x \times x = x^4$ 4 is exponent of X.

$a^m \times a^n$	$= a^{(m+n)}$	Where m and n are numbers
$\frac{a^m}{a^n}$	$= a^{(m-n)}$	Where m>n
$\frac{a^m}{a^n}$	$= a^{(\frac{1}{n-m})}$	Where m<n
$(a^m)^n$	$= a^{m \times n} = a^{mn}$	
$(p^a . q^b . r^c)^n$	$= p^{an} . q^{bn} . r^{cn}$	
$x^m \times y^m$	$= (x \times y)^m$	
$\frac{p^m}{q^m}$	$= (\frac{p}{q})^m$	

$x^3 = x \times x \times x$	$2^3 = 2 \times 2 \times 2 = 8$
$x^2 = x \times x$	$2^2 = 2 \times 2 = 4$
$x^1 = x$	$2^1 = 2$
$x^0 = 1$	$2^0 = 1$
$x^{-1} = \frac{1}{x^1}$	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
$x^{-2} = \frac{1}{x^2}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Problems :

1) Size of a bacteria is 10^{-4} cm. Convert into m.m.

Ans. $10^{-4} \times 10^{1} = 10^{-4+1} = \mathbf{10^{-3}}$

2) What is the value of 5 to power zero? (R. R. B. Exam.)

Ans. $5^{0} = 1$, ie $x^{0} = \mathbf{1}$

3) The value of $[7 - 5\ (3 - 2)^{-1}]^{-1}$ is_____ (Clerk Grade Exam.)

Ans: $x^{-1} = \frac{1}{x^{1}}$

$(7 - 5 \times 1^{-1})^{-1} = (7 - 5 \times \frac{1}{1})^{-1} = (7 - 5)^{-1}$

$= 2^{-1} = \frac{1}{2^{1}} = \mathbf{\frac{1}{2}}$

I. APPLICATION OF ALGEBRAIC EQUATIONS TO PRACTICAL PROBLEMS

1) If the combined age of father and the mother is 65 years, that of mother and son is 40 years and of father and son is 45 years, what is the age of the son?

Ans. : Let the father's age be 'F' years, mother's 'M' years and son's 'S' years.

$\therefore$ $F + M = 65$ ___ (1)

$M + S = 40$ ___ (2)

$F + S = 45$ ___ (3)

Adding the three equations, we get

$F + M + M + S + F + S = 65 + 40 + 45$

$2(F + M + S) = 150$

$F + M + S = \frac{150}{2} = 75$ ___ (4)

(4) - (1) gives, $(F + M + S) - (F + M) = 75 - 65 = 10$

$\therefore$ Age of son (S) = **10 years.**

2) In a theatre, the cost of a child ticket is $\frac{1}{3}$ of the cost of an adult ticket. If the cost of ticket for 3 adults, and 3 children is Rs. 60, the cost of an adult ticket is_____.

(Clerk Grade Exam.)

Ans. Let cost of an adult ticket be Rs.x

$\therefore$ Cost of child ticket $= \frac{1}{3}x$

$\therefore$ Cost of tickets for 3 adults and 3 children $= 3x + 3 \times \frac{1}{3}x$

$= \text{Rs. } 60$

$4x = \text{Rs. } 60$

$x = \frac{\text{Rs.}60}{4} = \mathbf{Rs.15}$

$\therefore$ Cost of an adult ticket = **Rs.15**

3) Father is 5 years older than mother and the mother's age now is thrice the age of the daughter. The daughter is now 10 years old. What was the father's age when the daughter was born?

Ans. Let the age of father be 'F' years and mother be 'M' years.

$F = M + 5$

$M = 3 \times 10 = 30$

$\therefore F = M + 5 = 30 + 5 = 35$

Father's age when daughter was born

= Present age - daughter's age

= 35 - 10

= **25 years.**

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6. PERCENTAGE

Percent, written as % or P.C., means for every hundred or per hundred. Percentage means calculation for every hundred. Thus a fraction expressed with 100 as its denominator is a percentage and the numerator of the fraction is a rate percent.

Changing a Percent to Fraction

To remove percent divide by 100.

eg. 1) $5\% = \frac{5}{100} = \frac{1}{20}$

To get the fraction in the simplest form divide both numerator and denominator by their H.C.F.

2) $15\% = \frac{15 \div 5}{100 \div 5} = \frac{3}{20}$

3) $33\frac{1}{3}\% = \frac{100\%}{3} = \frac{100}{3x100} = \frac{1}{3}$

Many times, when we are asked to find a certain percentage, we will find that it is not necessary to carryout the entire calculation. For example, to find 25% of a number multiply by ¼ instead of multiplying that number by $\frac{25}{100}\left(\text{ie, } \frac{25}{100} = \frac{1}{4}\right)$. The following shortcuts can be employed for finding certain percentage.

$10\% = \frac{10}{100} = \frac{1}{10}$	$6\frac{1}{4}\% = \frac{25}{4} \times \frac{1}{100} = \frac{1}{16}$
$20\% = \frac{20}{100} = \frac{1}{5}$	$8\frac{1}{3}\% = \frac{25}{3} \times \frac{1}{100} = \frac{1}{12}$
$25\% = \frac{25}{100} = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{25}{2} \times \frac{1}{100} = \frac{1}{8}$

$40\% = \frac{40}{100} = \frac{2}{5}$	$33\frac{1}{3}\% = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$
$50\% = \frac{50}{100} = \frac{1}{2}$	$66\frac{2}{3}\% = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3}$
$60\% = \frac{60}{100} = \frac{3}{5}$	$100\% = 100 \times \frac{1}{100} = 1$
$75\% = \frac{75}{100} = \frac{3}{4}$	

Convert into percent

To get percent multiply by 100.

eg:- 1) $\frac{1}{4} = \frac{1}{4} \times 100\% = \frac{100}{4}\% =$ **25%**

2) $\frac{4}{5} = \frac{4}{5} \times 100\% = \frac{400}{5}\% =$ **80%**

3) $\frac{1}{3} = \frac{1}{3} \times 100\% = \frac{100}{3}\% =$ **$33\frac{1}{3}$%**

Convert the following decimals into percent

1) $0.03 = \frac{3}{100} = \frac{3}{100} \times 100\% =$ **3%**

2) $0.8 = \frac{8}{10} = \frac{8}{10} \times 100\% =$ **80%**

3) $1.5 = \frac{15}{10} = \frac{15}{10} \times 100\% =$ **150%**

4) $0.505 = \frac{505}{1000} = \frac{505}{1000} \times 100\% =$ **50.5%**

Convert the following percent into decimal

1) $40\% = \frac{40}{100} = \mathbf{0.40}$

2) $6\% = \frac{6}{100} = \mathbf{0.06}$

3) $32.5\% = \frac{32.5}{100} = \mathbf{0.325}$

4) $125\% = \frac{125}{100} = \mathbf{1.25}$

Calculate each of the following

1) 40% of 230 $= 230 \times \frac{40}{100} = \mathbf{92}$

2) 25% of 50 $= 50 \times \frac{1}{4} = \mathbf{12.5}$

3) $8\frac{1}{3}$ of 1200 $= 1200 \times \frac{1}{12} = \mathbf{100}$

Find percentage

1) 210 marks out of 600 $= \frac{210}{600} \times 100\% = \mathbf{35\%}$

2) $12\frac{1}{2}$ marks out of 50 $= \frac{12.5}{50} \times 100 = \mathbf{25\%}$

Problems :

1) 5% of 25% of 200 is _____ . (U.S.S.Scholarship Exam.)

Ans: $\frac{5}{100} \times \frac{25}{100} \times 200 = \frac{5}{2} = \mathbf{2.5}$

2) 20% of 10% of $12\frac{1}{2}$ % of x = 3, find the value of x ?

Ans: $\frac{1}{5} \times \frac{1}{10} \times \frac{1}{8} \times x = 3$

$$\frac{x}{400} = 3$$

$$x = 3 \times 400 = \mathbf{1200}$$

3) 25% of 4 ÷ 4% of 25 = ?

Ans : $\frac{1}{4} \times 4 \div \frac{1}{25} \times 25 = 1 \div 1 = \mathbf{1}$

4) 20 is what percent of 200 ?

Ans: $200 \times \frac{x}{100} = 20$

$$2x = 20$$

$$x = \frac{20}{2} = \mathbf{10\%}$$

5) 20 is 10% of what ?

Ans: Let the number be x

10% of x = 20

$$\frac{1}{10} \times x = 20$$

$$x = 20 \times 10 = \mathbf{200}$$

6) K% of 75 = 30% of 50, find the value of K?

(S.B.I. P.O. Exam)

Ans: $\frac{k}{100} \times 75 = \frac{30}{100} \times 50$

$$k \times \frac{3}{4} = 15$$

$$k = 15 \times \frac{4}{3} = \mathbf{20}$$

7) 'A' travelled a distance of 20 km out of a total distance of 50 km. What percentage of the total distance he has yet to travel ?

(M.B.A. Entrance Exam.)

Ans : Total distance = 50 km.

Distance yet to travel = 50km - 20km = 30 km

Percentage of distance to be travelled $= \frac{30\text{km}}{50\text{km}} \times 100 = \mathbf{60\%}$

8) 12.5% of 192 = 50% of x, find the value of x ? (Bank P.O., Exam)

Ans: $\frac{1}{8} \times 192 = \frac{1}{2} \times x$

ie, $\frac{1}{2} \times x = \frac{192}{8} = 24$

$x = 24 \times 2 =$ **48**

9) One-third of a number is 13. Then 40% of the number is ___ .
(P.S.C. Divisional Accountant Test)

Ans : Let the number be x

$\frac{1}{3} \times x = 13$

$x = 13 \times 3 = 39$

40% of 39 $= \frac{40}{100} \times 39 =$ **15.6**

Number is **15.6**

10) If A gets 25% more than B and B gets 20% more than C, what will be the share of C out of a sum of Rs. 740 ?
(R.R.B. Exam)

Ans: Let the share of C be Rs. x.

Share of B = x + 20% of x = 1.2x

Share of A = 1.2x + 25% of 1.2x

$= 1.2x + \frac{1}{4} \times 1.2x$

$= 1.2x + 0.3x = 1.5x$

Total $= 1.5x + 1.2x + 1x = 740$

$3.7x = 740$

$x = \frac{740}{3.7} = 200$

$\therefore$ Share of C = **Rs. 200**

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7. PROFIT AND LOSS

Profit or Gain = Selling Price – Cost Price

Loss = Cost Price – Selling Price

If selling price is greater than cost price there will be profit and if cost price is greater than selling price there will be loss.

Selling price	= Cost price + profit
"	= Cost price – Loss
Cost price	= Selling price – profit
"	= Selling price + loss

If profit or loss is given in percent, first convert into amount and apply the formula to find selling price or cost price. Profit or loss is calculated based on cost price.

Problems :

1) Cost price Rs. 820, gain 15%, find selling price ?

Ans: Gain = Rs. 820 x $\frac{15}{100}$ = Rs. 123

Selling price = Rs. 820 + Rs. 123 = **Rs. 943**
(Cost price + profit)

or

Selling price = 115% of cost price

$= \frac{115}{100}$ x Rs. 820

= **Rs. 943**

2) If cost price is Rs. 800 and selling price is Rs. 840, find the percentage of profit ?

Ans: Profit = Selling price – Cost price

= Rs. 840 – Rs. 800 = Rs. 40

Percentage of Profit = $\frac{\text{Profit}}{\text{Cost price}} \times 100$

= $\frac{\text{Rs. } 40}{\text{Rs. } 800} \times 100 =$ **5%**

3) The price of sugar is increased by 20%. How much percentage consumption be reduced to keep the spending same.

(L.I.C. Asst. Grade Exam)

Ans: Let the consumption of sugar be x units

and the price per unit be 100

Then, total cost = 100x

Price per unit after 20% increase = 100 x 120% = 120

Consumption of sugar to keep the spending same $= \frac{100x}{120} = \frac{10}{12}x = \frac{5}{6}x$

Reduction in consumption $= \left(1 - \frac{5}{6}\right)x = \left(\frac{6}{6} - \frac{5}{6}\right)x = \frac{1}{6}x$

Percentage of reduction $= \frac{1}{6} \times 100 =$ **$16\frac{2}{3}$%**

4) A bookseller allows a discount of 10% on the marked price of a book and still makes a profit of 10%. A book costs him Rs. 9. Find the marked price of the book.

(P.S.C. Divisional Accountant Test)

Ans: Let the marked price of a book be x.

Selling price of book = $X \times 90\% = 0.9x$

90% of marked price = 110% of cost

$\frac{90}{100} \times X = \frac{110}{100} \times 9$

Marked price (X) = $\frac{110}{100} \times 9 \times \frac{100}{90} =$ **Rs. 11**

5) An article when sold at a gain of 5% yields Rs. 15 more than when sold at a loss of 5%. what is its cost price ?

(R.R.B. Exam.)

Ans: Let the cost price be Rs. x (ie 100%x)

Selling price at 5% gain = 105% of x

Selling price at 5% loss = 95% of x

105% of x – 95% of x = Rs. 15

10% of x = Rs. 15

$\frac{1}{10} \times X =$ Rs. 15

$X =$ Rs. 15 x 10 = **Rs. 150**

Discount may be offered either in cash or in goods. When discount is offered in cash it is generally expressed as a percentage of the marked price (M.P.) on goods.

Selling price = Marked price – Discount.

6) A shopkeeper marks his goods 20% above cost price, but allows 10% discount for cash. His net profit is ____ .

(Clerk Grade Exam)

Ans: Let the cost price be Rs. 100

Market price = 100 + 20 = 120

Discount = 120 × 10% = 12

Selling price = 120 – 12 = 108

Profit = 108 – 100 = 8

Profit percent = $\frac{8}{100} \times 100 =$ **8%**

7) The cost of a table and a chair is Rs. 1250. If the table cost 4 times as much as a chair, what is the cost of a table ?

Ans: Let the cost of a chair be x

∴ Cost of table = 4x

x + 4x = Rs. 1250

5x = Rs. 1250

$x = \frac{\text{Rs. } 1250}{5} =$ Rs. 250

Cost of the table = 4 × 250 = **Rs. 1000**

8) When the price of T.V. set was increased by 30%, the number of T.V. sets sold decreased by 20%. What was the effect on the sales ? (Bank P.O. Exam.)

Ans: Suppose the price of T.V. set is Rs. 100

and number of T.V. sets sold is 100.

$\therefore$ Total revenue received = Rs.100 × 100 = Rs.10,000

When price increase by 30% price of T.V. set = Rs. 130

Then number of T.V. sets sold = 100 × 80% = 80

Total revenue received in this case = Rs. 130 × 80

= Rs. 10,400

$$\text{Sales per T.V.} = \frac{\text{Rs.10400}}{100} = \text{Rs.104}$$

Profit = 104 – 100 = Rs.4

$$\text{Effect on the sales} = \frac{4}{100} \times 100 = \textbf{4\% increase}$$

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8. AVERAGE

$$\text{Average} = \frac{\text{Sum of values of the set}}{\text{Number of values in the set}}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

Sum of the values = Average × number

$\Sigma x = \bar{x} \times n$

Problems :

1) Find the average of 3.5, 8.3 and 6.2 ?

Ans: Average ($\bar{x}$) $= \frac{\Sigma x}{n} = \frac{3.5 + 8.3 + 6.2}{3}$

$= \frac{18}{3} = \mathbf{6}$

2) The average of 4 numbers is 12.5. Three of them are 7, 17 and 25. What is the 4th number ? (U.S.S.Scholarship Exam)

Ans: Average ($\bar{x}$) = 12.5, n = 4

Sum (Σx) = $\bar{x} \times n$

= 12.5 × 4 = 50

Sum of three numbers = 7 + 17 + 25 = 49

4th number = 50 - 49 = **1**

3) The sum of seven numbers is 235. The average of the first three is 23 and that of the last three is 42. The fourth number is ____ . (Clerk Grade Exam)

Ans: Sum of seven numbers = 235

Sum of first 3 numbers = 23 x 3 = 69

Sum of last 3 numbers = 42 x 3 = 126

Sum of 6 numbers excluding fourth number = 69 + 126 = 195

Fourth number = 235 - 195 = **40**

4) The average age of 12 children is 20 years. If the age of one more child is added the average decreases by 1, what is the age of the child added later ? (B.S.R.B. Exam)

Ans: Sum of the age of 12 children = 20 x 12 = 240

Average age of 13 children = 20 - 1 = 19

Sum of the age of 13 children = 19 x 13 = 247

Age of the child added = 247 - 240 = **7 years**

5) The average wt. of a class of 33 students has been workout to be 21 kg. For one student, however, wt. of 16 kg was recorded in place of 19 kg. The true average wt. per student is approximately _____ .

(Modern Food Industries Ltd., Selection of Management Trainees.)

Ans: Total weight of 33 students = Average × number

= 21 kg x 33 = 693 kg.

Correct total = 693 + 19 - 16 = 696 kg.

$$\text{True average wt.} = \frac{696 \text{ kg.}}{33} = \textbf{21.09 kg}$$

6) The average age of A, B & C is 20. If the average of the age of B & C is 22 years, what is the age of A in years ?

(S.B.I. P.O. Exam)

Ans: The average age of A, B & C = 20 years

Sum of their age = 20 x 3 = 60 years

The average of B & C = 22 years

Sum of their age = 22 x 2 = 44 years

∴ Age of A = 60 - 44 = **16 years**

7) The average of 'a' and 'b' is 26. The average of 'b' and 'c' is 21. The average of 'a' and 'c' is 30. What are the values of a,b and c?

Ans: Total of 'a' and 'b' = 26 x 2 = 52

Total of 'b' and 'c' = 21 x 2 = 42

Total of 'a' and 'c' = 30 x 2 = 60

$$\begin{aligned} a + b &= 52 \\ b + c &= 42 \\ a + c &= 60 \\ \hline 2a + 2b + 2c &= 154 \end{aligned}$$

$2(a+b+c) = 154$

$a + b + c = 77$ (a+b =52)

$52 + c = 77$

$c = 77 - 52 = \mathbf{25}$

$a = 60 - 25 = \mathbf{35}$

$b = 52 - 35 = \mathbf{17}$

8) The average age of 32 students is 10 years. If the teacher's age is also included, the average increases by one year. What is the age of the teacher ? (Bank P.O. Exam)

Ans: Total age of 32 students = 32 x 10 = 320

Total age of 32 students and teacher = 33 x 11 = 363

Age of the teacher = 363 - 320

= **43 years**

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9. RATIO AND PROPORTION

A. Ratio

B. Proportion

C. Variation

D. Application of Ratio and Proportion to practical problems

A. RATIO

Ratio is the relationship between two elements of the same kind. Ratio between **a** and **b** is expressed by a fraction $\frac{a}{b}$, where **a** is called the antecedent and **b** the consequent, **a & b** are also called terms of the ratio. A ratio is usually given in the smallest possible whole number.

eg: $\frac{10}{30} = 10 : 30 = 1:3$

Inverse Ratio :- Product of a : b and b : a is 1.

ie, $\frac{a}{b} \times \frac{b}{a} = 1$

Compound Ratio :- Product of antecedent : Product of Consequent.

4 : 3 and 5 : 10 can be written as

$4 \times 5 : 3 \times 10 = 20 : 30 = 2 : 3$

Ratio in the simplest form

To express a ratio in the simplest form, divide both the antecedent and consequent by their H C F.

eg: 1) Express 36 : 54 in the simplest form

H C F $= 2 \times 3 \times 3 = 18$

Divide the terms by H C F.

$36 : 54 = \frac{36}{18} : \frac{54}{18} = 2 : 3$

2	36, 54
3	18, 27
3	6, 9
	→ 2, 3

Alternative Method :

The indivisible numbers, when factorising, are ratio in the simplest form.

ie, 36 : 54 = 2 :3

2) Express 25 paise to 1 rupee in the simplest form ?

The terms in the ratio should be in same unit.

5	25, 100
5	5, 20
	→ 1, 4

25 : 100 = 1 : 4

Conversion of ratios in the simplest form if elements are fractions with same denominators :

Ex: 1) $\frac{3}{8} : \frac{5}{8} = 3 : 5$

2) $\frac{5}{7} : \frac{3}{7} : \frac{1}{7} = 5 : 3 : 1$

(No need of writing denominators in the ratio.)

Conversion of ratios in the simplest form if elements are fractions with different denominators:

Ex: 1) $\frac{3}{5} : \frac{3}{4}$

(First equalise the denominators. For this find L.C.M.)

L C M of 5 and 4 $= 5 \times 4 = 20$

$\frac{3 \times 4}{5 \times 4} : \frac{3 \times 5}{4 \times 5} = \frac{12}{20} : \frac{15}{20} = 12 : 15 = 4 : 5$

Problems :

1) Ali spends Rs. 600 in one month out of his salary of Rs. 1000. Find the following ratios?

a) Salary : Expenses

b) Salary : Savings

c) Expenses : Savings

Salary = Rs. 1000
Expenses = Rs. 600
Savings = Rs. 1000 - Rs. 600
= Rs. 400

Ans: a) Salary : Expenses = 1000 : 600 = **5 : 3**

b) Salary : Savings = 1000 : 400 = **5 : 2**

c) Expenses : Savings = 600 : 400 = **3 : 2**

2) A profit of Rs. 7,200 is to be distributed among A, B and C in the ratio of 3 : 2 : 1. The difference between the amount received by A and C is _____ . (Bank P.O. Exam)

Ans: A : B : C = 3 : 2 : 1

Sum of the ratio = 3 + 2 + 1 = 6

A's Share = Rs. $7200 \times \frac{3}{6}$ = Rs.3600

C's Share = Rs. $7,200 \times \frac{1}{6}$ = Rs.1200

Difference = Rs.3600 _ Rs.1200

= **Rs. 2400**

OR

Difference between the amount received by A and C = $\left(\frac{3}{6} - \frac{1}{6}\right)$ of Rs. 7200

= $\frac{2}{6} \times$ Rs.7200

= **Rs. 2400**

3. The prices of a T V set and a radio are in the ratio 7:2. If a T V set costs Rs. 3500 more than a radio, what is the price of a radio ? (Bank P.O. Exam)

Ans: Suppose price of T V set is 7x and radio is 2x.

Difference of price = 7x _ 2x = Rs.3500

5x = Rs.3500

x = $\frac{\text{Rs.3500}}{5}$

= Rs.700

Price of radio = 2x = 2 × Rs.700 = **Rs.1400**

4) A bag contains 25 paise, 10-paise and 5-paise coins in the ratio 1:2:3. If their total value is Rs.30, the number of 5-paise coins is ___ . (Clerk's grade Exam)

Ans: Let the numbers of 25-paise, 10-paise and 5-paise coins are x, 2x and 3x, respectively.

Then, 25 × X + 10X 2X + 5 × 3X = Rs.30

25x + 20x + 15x = 3000 paise

60x = 3000

x = $\frac{3000}{60}$ = 50

Number of 5-paise coins = 3x = 3 x 50 = **150**

B. PROPORTION

The equality of two ratios is called a proportion. In other words, if four quantities are so related that the ratio of the first to the second is equal to the ratio of the third to the fourth, then the four quantities are said to be in proportion.

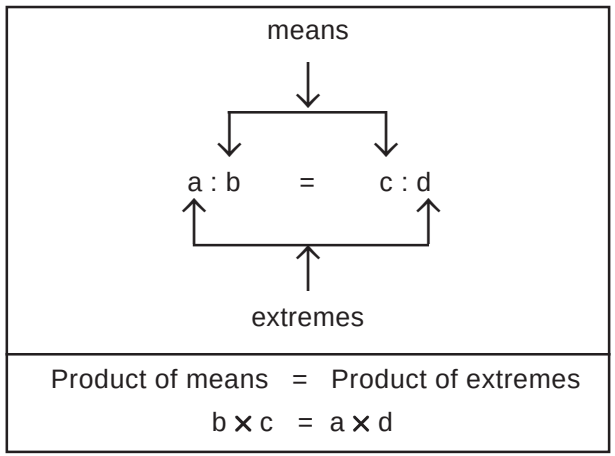
Ex: A train moving at 50km per hour will take 4 hours to travel a distance of 200 km. Another train moving at 40km per hour will take 5 hours to cover the same distance.

Then, $50 : 40 = 5 : 4$

$a : b = c : d$ or $a : b :: c : d$

then, a, b, c, d are in proportion.

$$\frac{a}{b} = \frac{c}{d} = a : b = c : d$$



Ex: 1. Find the value of x in $5 : 10 = 15 : x$

Ans: Product of extremes = product of means

$$5 \times x = 10 \times 15$$

$$x = \frac{150}{5} = \mathbf{30}$$

2. $24 : X = 15 : 35$, find the value of X ?

Ans: product of means = product of extremes

$$X \times 15 = 24 \times 35$$

$$X = \frac{24 \times 35}{15} = \mathbf{56}$$

3. $\frac{1}{3} : \frac{1}{4} :: X : \frac{1}{2}$, $X = ?$

Ans: $\frac{1}{4} \times X = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$$X = \frac{1}{6} \times \frac{4}{1} = \mathbf{\frac{2}{3}}$$

Problems :

1. Find the value of x in $3:X = 4:5$

(M.B.A. Entrance Exam)

Ans: $3 : X = 4 : 5$

$X \times 4 = 3 \times 5 = 15$

$X = \frac{15}{4} = \mathbf{3\frac{3}{4}}$

2. If A : B = 3 : 4 and B : C = 5 : 6, then A : B : C equals _____.

(Clerk's Grade Exam)

Ans: A : B : C

3 : 4

5 : 6

The Combined Proportion is obtained by

$3 \times 5 : 4 \times 5 : 4 \times 6$

= **15 : 20 : 24**

3. If A : B = 7 : 6 and B : C = 3 : 4, then A : B : C equals ___.

(Inspectors of Central Excise etc. Exam)

Ans: A : B : C

7 : 6

3 : 4

Combined proportion = $7 \times 3, 6 \times 3, 6 \times 4$

= 21 : 18 : 24

= **7 : 6 : 8**

4. If a + b : a - b = 5 : 2, find the value of b : a.

(Calicut university B.Com Exam.)

Ans :

$$a + b : a - b = 5 : 2$$

Product of means = Product of extremes

$$5(a-b) = 2(a+b)$$

$$5a-5b = 2a+2b$$

$$5a-2a = 2b + 5b$$

$$3a = 7b$$

$$\frac{a}{b} = \frac{7}{3}$$

$$a : b = 7 : 3$$

$$b : a = \mathbf{3 : 7}$$

C. VARIATION

Direct Variation

Two quantities x and y are said to vary directly with each other, if they increase or decrease together in such a manner that the ratio of their corresponding values remains constant.

When x and y vary directly, we say that x is proportional to y. ($x \alpha y$).

If $x \alpha y$, $x = ky$ or $\frac{x}{y} = k$ Where, k is a Positive Constant number.

Examples for direct variation :

1) When demand increase price will increase.

2) When speed increase distance travelled per hour will increase.

Inverse Variation

Two quantities x and y are said to vary inversely with each other if an increase in x causes a decrease in y and conversely in such a manner that the product of their corresponding values remains constant.

ie, $X \alpha \frac{1}{y}$ or $X \times y = k$ or $X = k\frac{1}{y}$

Examples for inverse variation:

1) When supply increase price will decrease.

2) When speed increase time taken to travel will decrease.

Problem :

If x varies directly as y and inversely as z and x = 10, when y = 12, z = 6, then the value of x, when y = 18 and z = 5 is ___ . (C.D.S. Exam)

Ans: x varies directly as y and inversely as z.

Then, $x \alpha \frac{y}{z}$, $x = \frac{ky}{z}$, where k is a constant.

Given x = 10 when y = 12 and z = 6

$$\therefore 10 = \frac{k \times 12}{6} = k \times 2,\ k \times 2 = 10,\ k = \frac{10}{2} = 5$$

Thus $x = \frac{5y}{z}$. Finally, when y = 18 and z = 5

$$x = \frac{5 \times 18}{5} = \mathbf{18}$$

D. APPLICATION OF RATIO AND PROPORTION TO PRACTICAL PROBLEMS.

1. The ages of A & B are in the ratio of 6 : 5 and sum of their ages is 44 years. What will be the ratio of their ages after 8 years ?

(Bank P.O. Exam)

Ans :

$$A : B = 6 : 5 = 6x : 5x$$

Sum of the ages = $6x + 5x = 44$

$$11x = 44$$

$$x = \frac{44}{11} = 4$$

$$A = 6x = 6 \times 4 = 24$$

$$B = 5x = 5 \times 4 = 20$$

After 8 years, A : B = 24 + 8 : 20 + 8

= 32 : 28

= **8 : 7**

2. Six years ago the ratio of the ages of Kamal and Suresh was 6:5, four years hence the ratio of their ages will be 11 :10 what is Suresh's age at present ? (S.B.I. P.O. Exam)

Ans: Six years ago → Kamal : Suresh = 6 : 5 = 6x : 5x

$$\text{ie,} \quad \frac{\text{Kamal}}{\text{Suresh}} = \frac{6x}{5x}$$

$$\text{At present} \rightarrow \quad \frac{\text{Kamal}}{\text{Suresh}} = \frac{6x + 6 + 4}{5x + 6 + 4} = \frac{11}{10} \text{ (given)}$$

Cross multiplying, we have $10(6x + 10) = 11(5x + 10)$

$$60x + 100 = 55x + 110$$

$$60x - 55x = 110 - 100$$

$$5x = 10$$

$$x = \frac{10}{5} = 2$$

Suresh's age at present = 5x + 6

= 5 x 2 + 6 = 10 + 6 = **16 years**

3. The ratio of two numbers is 3 : 4. If 5 is added to both the numbers, their ratio becomes 4 : 5. What is the value of the largest number ?

Ans: The ratio of two numbers is 3 : 4

Let 'x' be the common factor among them.

Then, the numbers are 3x and 4x

On adding 5 to both the numbers, they become

(3x + 5) and (4x + 5) respectively.

Their new ratio = 4 : 5

$$\therefore \frac{3x + 5}{4x + 5} = \frac{4}{5}$$

By cross - multiplication, $4(4x+5) = 5(3x+5)$

$$16x + 20 = 15x + 25$$

$$16x - 15x = 25 - 20$$

$$x = 5$$

The numbers are (3x5) or 15 and (4x5) or 20

The largest number = **20**

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10. TIME AND DISTANCE

Time Required for a journey (Time Taken)	$= \frac{\text{Distance Travelled}}{\text{Speed}}$
Distance Travelled = Speed × Time taken	
$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$	

Unit of Speed = km / hr. or m / sec

$$1 \text{ km/hr.} \frac{1000\text{m}}{60 \times 60 \text{ sec.}} = \frac{5}{18} \text{ m/sec.}$$

km = Kilo metre
hr. = hour
m. = metre
sec.= second

Problems :

1. If a man walks a distance of 16 km. in 2 hrs. find his speed of walking ?

Ans: $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{16 \text{ km.}}{2\text{hrs.}}$

= **8 km/hr.**

2. A train, 700 metres long, is running at the speed of 72 km/hr. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is ____. (N.D.A. Exam)

Ans: Speed of the train 72 km/hr. $= 72 \times \frac{5}{18}$ m/sec.

= 20 m/sec.

Distance travelled in 1 minute = 20 m/sec. × 60 sec.

= 1200 m.

Length of tunnel = 1200 m - 700 m

= **500 m.**

3. A train, 120 metres long is running at a uniform speed of 90km/hr. Find the time taken by it to cross a bridge 605 metres long ?

Ans: Total distance = 120 m + 605 m = 725 m

Speed = 90 km/hr = $90 \times \frac{5}{18}$ m/sec. = 25 m/sec.

Time required to travel 25m = 1 sec.

Time required to travel 1m = $\frac{1}{25}$ sec.

Time required to travel 725m = $\frac{1}{25}$ x 725 sec. = 29 sec.

ie, Time required for the train to cross the bridge = **29 sec.**

4. If the speed of a train is 92.4 km/hour, then how many metres are covered by it in 20 minutes ? (Bank P.O. Exam.)

Ans: Speed of the train = 92.4 km/hr = 92400m. in 60 minutes.

Distance covered in 1 minute = $\frac{92400}{60}$ metre.

Distance covered in 20 minutes = $\frac{92400}{60} \times 20$ = **30800 metres.**

5. A train travels at an average speed of 80 km per hour for $2\frac{1}{2}$ hrs. and then travels at a speed of 100km/hr. How far did the train travel after 6 hours.

(LIC Asst. Grade Exam.)

Ans: Distance travelled by the train in the first $2\frac{1}{2}$ hours = 80 km/hr x 2.5 hrs.

= 200 km

Distance travelled in next $3\frac{1}{2}$hrs. = 100 km/hr x 3.5 hrs.

[3.5 hrs = 6 hrs - 2.5 hrs] = 350 km

Total distance travelled by the train = 200 + 350

= **550 km**

6) A car can finish a certain journey in 10 hrs at the speed of 48 km per hour. By how much should its speed be increased so that it may take only 8 hours to cover the same distance ?

Ans: Total distance = 10hrs. x 48 km/hr = 480 km

Speed of the car if it takes only 8 hours = $\frac{480 \text{ km}}{8 \text{ hrs}}$ = 60 km/hr

Increase in speed = (60 - 48) km/hr = **12 km/hr**

7. Ram travels at the rate of 3 km/hr. He reaches 15 minutes late. If he travels at the rate of 4 km/hr, he reaches 15 minutes earlier. The distance Ram has to travel is _____ . (CDS Exam)

Ans: Let the distance travelled by Ram be x km

Time taken if he travels at 3 km/hr = $\frac{x}{3}$ hrs

Time taken if he travels at 4 km/hr = $\frac{x}{4}$ hrs

Then, $\frac{x}{3} - \frac{15}{60} = \frac{x}{4} + \frac{15}{60}$

$\frac{x}{3} - \frac{x}{4} = \frac{15}{60} + \frac{15}{60}$

$\frac{4x - 3x}{12} = \frac{30}{60} = \frac{1}{2}$

$x = \frac{1}{2} \times 12$ = **6 km**

8. At 10 A.M. one train leaves Mumbai for Delhi and another leaves Delhi for Mumbai. If their respective speeds are 120kmph and 100 kmph. at what time will they meet, presuming the distance between Mumbai and Delhi is 1540 kms.

Ans: The two trains are running in opposite directions.

Their relative speed = 120 + 100 = 220 km/hr.

The distance between Mumbai and Delhi = 1540 kms.

$\therefore$ the trains will meet after $\left(\frac{1540}{220}\right)$ hrs or 7 hrs.

Since both the trains have started at 10 a.m. they will meet 7 hours after 10 a.m. ie, at 5 p.m.

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11. TIME AND WORK

If a man can do a piece of work in 4 days, it is clear that he can do 1/4 of the work in one day. Again, if a man can do 1/4 of the work in one day, he will take 4 days to finish the work.

To solve problems relating time and work we apply unitary method. The method of finding first the value of one quantity (unit) from the value of the given quantities and then the value of required quantities is called the **Unitary Method.**

Problems :

1. 15 persons can fill 35 boxes in 7 days. How many persons can fill 65 boxes in 5 days ?

(Bank P.O., Exam.)

Ans: Mandays required to fill 35 boxes = (15 x 7) days

Mandays required to fill 1 box $= \frac{15 \times 7}{35} = 3$ days

No. of persons required to fill 65 boxes in 5 days $= \frac{3 \times 65}{5} =$ **39**

2. If 5 men take an hour to dig a ditch, how long should it take 12 men to dig the same ditch ?

(LIC Asstt. Grade II Exam.)

Ans: Total man hours required to dig the ditch

= 5 men × 1 hr.

= 5 man hours

Time taken, if 12 men do the work $= \frac{5}{12}$ hrs.

$= \frac{5}{12} \times 60$ minutes

= **25 minutes**

3. If 12 boys can earn Rs. 240 in 5 days, how many boys can earn Rs. 420 in 21 days ?

(PSC Divisional Accountant Test)

Ans: 12 boys earn in 5 days = Rs. 240

$\therefore$ 1 boy earns in 1 day $= \frac{\text{Rs. } 240}{12 \times 5}$

= Rs. 4

No. of boys who earn Rs. 420 in 21 days $= \frac{\text{Rs. } 420}{21 \times \text{Rs.} 4}$

= **5**

4. 12 men complete a work in 18 days. Six days after they had started working, 4 more men joined them. How many days will all of them take to complete the remaining work ?

(SBI P.O, Exam)

Ans: Mandays required to complete the work

= 12×18 = 216

Six day's work done

by 12 men = 12 × 6 = 72 mandays

Remaining work = 216 -72 = 144 mandays

Number of days required to complete the remaining work by 16 men $= \frac{144 \text{ mandays}}{16 \text{ men}}$

= **9 days**

5. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same job. How long should it take worker A and B working together to do the same job ?

(LIC Asstt. Grade Exam)

Ans: A will do $\frac{1}{8}$ th of the job in 1 hour.

B will do $\frac{1}{10}$ th of the job in 1 hour.

If they work together they will take

$\left(\frac{1}{8}+\frac{1}{10}\right)$ th of the job in 1 hour.

$= \frac{5+4}{40}$ (LCM of 8 and 10 is 40)

$= \frac{9}{40}$ th of the job in 1 hour.

Total time taken by them to do the same job $= \frac{1}{9/40} = \frac{1 \times 40}{9} = \frac{40}{9}$ hrs.

$=$ **4.44 hrs.**

6) A tap can fill a water tank in 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, the time (in hours) to fill the tank is _____.

(Clerks' Grade Exame)

Ans: In one hour the incoming water tap can fill $\frac{1}{8}$ th of the tank capacity

In one hour the outgoing water tap can empty $\frac{1}{16}$ th of the tank capacity

If both the taps are opened simultaneously, the tank can be filled $\left(\frac{1}{8}-\frac{1}{16}\right)$ th of its capacity in one hour.

$= \frac{1}{8}-\frac{1}{16} = \frac{2-1}{16} = \frac{1}{16}$ th in one hour.

Time to fill the tank $= \frac{1}{1/16} = \frac{1 \times 16}{1} =$ **16 hours**

7) There are two taps 1 and 2 in a tank. If both the taps are opened the tank will be drained in 20 minutes. If tap 1 is closed and tap 2 is opened the tank will be drained in 30 minutes. If tap 2 is closed and tap 1 is open how long will it take to drain the tank ?

(LIC Asst.Grade Exam)

Ans: If both the taps are opened, water flowers

per minute $= \frac{1}{20}$ th of total water

If tap 2 is opened and tap 1 is closed, water flows

per minute $= \frac{1}{30}$ th of total water

If tap 2 is closed and tap 1 is opened, water flows

per minute $= \left(\frac{1}{20}-\frac{1}{30}\right)$ th of total water

$= \frac{3-2}{60} = \frac{1}{60}$ "

Time taken to drain the tank $= \frac{1}{1/60} = \frac{60}{1}$

$=$ **60 minutes**

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ABBREVIATIONS

B. S. R. B	- Banking Service Recruitment Board
C. D. S	- Combined Defence Services
C. A.	- Chartered Accountant
I. C. W. A	- Institute of Cost and Works Accountant
I. T. I	- Industrial Training Institute
L.I.C	- Life Insurance Corporation
M.B.A	- Master of Business Administration
M.G.U	- Mahatma Gandhi University
N.D.A	- National Defence Academy
P.S.C	- Public Service Commission
P.O. Exam	- Probationary Officers Examination
R. R. B	- Railway Recruitment Board
S. B. I	- State Bank of India

12. MIXTURES

$$\text{Average Price of Mixture} = \frac{\text{Total Price}}{\text{Total Quantity}}$$

Problems :

1. Subhash bought 20kg. of tea at the rate of Rs. 30 per kg and 30 kg at the rate of Rs. 25 per kg. He mixed the two and sold the mixture at the rate of Rs. 22.50 per kg. What was his loss in this transaction ? (Bank P.O. Exam)

Ans: Cost of 20 kg. of tea = 20kg × Rs. 30 = Rs. 600

Cost of 30 kg of tea = 30 kg × Rs. 25 = Rs. 750

Total cost of (20 + 30) kg of tea = Rs. 600 + Rs. 750

= Rs. 1350

Sales value of mixture = 50 kg × Rs. 22.50

= Rs. 1125

Loss = Rs. 1350 - Rs.1125

= **Rs. 225**

2. In a mixture of 50 litres, milk and water are in the ratio of 3:2. How much water should be added to make the ratio of the two equal ? (R.R.B. Exam)

Ans: Total quantity of mixture = 50 litres

Ratio of milk and water = 3 : 2

Sum of the ratio = 3 + 2 = 5

Quantity of milk = $50 \times \frac{3}{5}$ = 30 litres

Quantity of water = $50 \times \frac{2}{5}$ = 20 litres

Water to be added to make the ratio of the two equal = 30 - 20 = **10 Litres**

3. In a mixture of 60 litres, ratio of milk and water is 2 :1. If the ratio of the milk and water is to be 1 : 2, then the quantity of water (in litres) to be further added is _____. (N.D.A. Exam)

Ans: Ratio of milk and water in the mixture of 60 litres = 2 : 1

Quantity of milk = $60 \times \frac{2}{3}$ = 40 litres

Quantity of water = $60 \times \frac{1}{3}$ = 20 litres

If the ratio of milk and water is to be 1:2, the quantity of water should be 80 litres. (ie, 2 × 40).

∴ Quantity of water to be added = 80 - 20 = **60 litres**

4. 15 litres of a mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed in it, the percentage of alcohol in the new mixture will be _____. (Clerk's Grade Exam)

Ans: Quantity of alcohol in the mixture = $15 \times \frac{20}{100}$ = 3 litres

Total Quantity of the mixture after mixing 3 litres.

= 15 + 3 = 18 litres

Percentage of alcohol in the new mixture

= $\frac{3}{18} \times 100$ = **16** $\frac{\mathbf{2}}{\mathbf{3}}$ **%**

5. A bottle contains 12 litres of acid of 75% concentration. Find out the quantity of water to be added to dilute the acid to 50% concentration ? (LIC Asstt. Grade II Exam)

Ans: Total quantity of acid in the bottle = 12 litres

Quantity of pure acid = 12 x 75% = 9 litres

Quantity of water = 12 - 9 = 3 litres

Quantity of water to be added = 9 - 3 = **6 litres**

(50% concentration means pure acid and water are in equal quantity)

6. Two varieties of tea, one costing Rs. 15 per kg. and another costing Rs. 10 per kg are mixed together. If the average cost of the mixture is Rs. 13 per kg, find the ratio of the mixture ?

(Pre-Degree Commerce (M.G.U.))

Ans: Let the quantity of first variety of tea be x and second variety be y.

Total cost = Rs.15x + Rs.10y

Average cost = $\frac{\text{Total cost}}{\text{Total quantity}}$

$\frac{Rs.15x+Rs.10y}{x+y}$ = Rs.13

By cross multiplication,

we get 15x + 10y = 13 (x+y)

15x+10y = 13x + 13y

ie, 15x - 13x = 13y - 10y

2x = 3y

$\frac{x}{y} = \frac{3}{2}$

Ratio of the mixture x : y = **3:2**

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13. SQUARES AND SQUARE ROOTS

A. SQUARES

The square of a number is that number 'raised to the power 2'.

Ex: Square of 3 is $3^2 = 3 \times 3 = 9$

Perfect Squares

Numbers which have whole numbers as their Square roots are called perfect squares. Perfect squares are 1, 4, 9, 16, 25, 36, 49, 64 etc.

- A number ending in 2,3,7 or 8 is never a perfect square.
- The number of zeros in the end of a perfect square is never odd.
- Squares of even numbers are always even.
- Squares of odd numbers are always odd.

Square of a Fraction :

Square of a fraction = $\frac{\text{Square of its numerator}}{\text{Square of its denominator}}$

Ex: 1. $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$, 2. $\left(5\frac{3}{4}\right)^2 = \frac{23^2}{4^2} = \frac{529}{16}$

Square of Decimals:

Ex: 1. $(1.3)^2 = 1.3 \times 1.3 = 1.69$

2. $(1.05)^2 = 1.05 \times 1.05 = 1.1025$

Number of decimal places in the square will be double. ie, two decimal places in number becomes four decimal places in square.

Ex: $(1)^2 = 1$ $(0.01)^2 = 0.0001$

$(0.1)^2 = 0.01$ $(0.001)^2 = 0.000001$

Study the following carefully :

Perfect squares of numbers upto 25	
Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

$1^2 = 1 = 1$
$2^2 = 1 + 2 + 1 = 4$
$3^2 = 1 + 2 + 3 + 2 + 1 = 9$
$4^2 = 1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$

$1^2 = 1 = 1$
$2^2 = 1 + 3 = 4$
$3^2 = 1 + 3 + 5 = 9$
$4^2 = 1 + 3 + 5 + 7 = 16$
$5^2 = 1 + 3 + 5 + 7 + 9 = 25$

$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$4^2 + 5^2 + 20^2 = 21^2$

$11^2 = 1\ 2\ 1$
$111^2 = 1\ 2\ 3\ 2\ 1$
$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$
$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

$(n + 1)^2 - n^2 = (n + 1) + n$ or $2n + 1$
$9^2 - 8^2 = 9 + 8 = 17$
$17^2 - 16^2 = 17 + 16 = 33$
$100^2 - 99^2 = 100 + 99 = 199$, etc.

eg: The difference between the squares of two consecutive numbers is 35. The numbers are ____.
(Hotel Management Deploma Course Entrance Exam.)

$(N+1)+n = 35$
$2n = 35 - 1 = 34$
$n = 34 \div 2 = 17$
The numbers are 17 and 18.

Square of numbers ending 5 as unit's place :

Ex:- $5 \times 5 = 25$
$15 \times 15 = 225$
$35 \times 35 = 1225$
$95 \times 95 = 9025$

$15 \times 15 = 1 \times 2$ (hundreds) $+ 5^2 = 225$
$35 \times 35 = 3 \times 4$ (") $+ 5^2 = 1225$
$95 \times 95 = 9 \times 10$ (") $+ 5^2 = 9025$

In square, last two digits will be 25. To get the previous digit(s), multiply the digit(s) before the last digit 5 with next counting number.

Ex: $35 \times 35 = 1225$
Last two digits = 25
Previous digits are $3 \times 4 = 12$

Square of numbers ending 1 as unit's place :

Ex: $1 \times 1 = 1$
$11 \times 11 = 121$
$41 \times 41 = 1681$
$101 \times 101 = 10201$

In square, last digit will be 1. Tens' place digit will be sum of the digits excluding 1. Left end digit or digits will be product of the digits excluding 1 plus remainder of addition if any.

1) $101 \times 101 = 10201$
Last digit = 1
Tens' place digit = $10 + 10 = 20$
Left end digits = $10 \times 10 + 2$
$= 100 + 2 = 102$

2) $41 \times 41 = (4 \times 4), 4 + 4), 1 = 1681$

B. SQUARE ROOTS

The Square root of a number is that number which when multiplied by itself is equal to the given number. '$\sqrt{\ }$' is the symbol of square root.

Ex: $\sqrt{100} = 10$, $\sqrt{64} = 8$, $\sqrt{400} = 20$, $\sqrt{2.25} = 1.5$

The number of decimal places or zeros in the square root will be half.

$\sqrt{10000} = 100$	$\sqrt{62500} = 250$	$\sqrt{1} = 1.000$
$\sqrt{100} = 10$	$\sqrt{625} = 25$	$\sqrt{2} = 1.414$
$\sqrt{1} = 1$	$\sqrt{6.25} = 2.5$	$\sqrt{3} = 1.732$
$\sqrt{0.01} = 0.1$	$\sqrt{0.0625} = 0.25$	$\sqrt{4} = 2.000$
$\sqrt{0.0001} = 0.01$	$\sqrt{0.000625} = 0.025$	$\sqrt{5} = 2.236$

Square root of a perfect Square by Factorisation method

Ex: 1. $\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5} = \sqrt{5^2} \times \sqrt{5^2}$

$= 5 \times 5 = 25$

2) $\sqrt{\frac{9}{121}} = \frac{\sqrt{3 \times 3}}{\sqrt{11 \times 11}} = \frac{3}{11}$

$\sqrt{a} = a^{1/2}$

$\sqrt{a^2} = a$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

3) $\sqrt{4900} = \sqrt{2 \times 2 \times 5 \times 5 \times 7 \times 7}$

$= \sqrt{2^2 \times 5^2 \times 7^2}$

$= 2 \times 5 \times 7$

$= 70$

2	4900
2	2450
5	1225
5	245
7	49
	7

OR

100	4900
7	49
	7

$\sqrt{100 \times 7^2} = 10 \times 7 = 70$

Square root of a number by division method

In this case, first we form groups consisting of two digits from right to left. In the case of decimal numbers, grouping starts from decimal point to either side.

146.8944 can be grouped as 1‘46.89 ‘44

Ex: 1. $\sqrt{625} = ?$

$\sqrt{625} = 25$

(Double of 2 = 4)

		2 5
	2	6‘25
		4
4	5	225
		225
		000

2) Find the square root of 146.8944.

$\sqrt{146.8944} = 12.12$

$\sqrt{14689.44} = 121.2$

$\sqrt{1.468944} = 1.212$

Double of 1 = 2

Double of 12 = 24

Double of 121 = 242

		1 2 . 1 2
	1	1‘46.89 ‘44
		1
22		46
		44
241		289
		241
2422		4844
		4844
		0000

Problems

1. The numerals possible in the units place of a perfect square are _____. (N.D.A. Exam)

   Ans: 0, 1, 4, 5, 6, 9

2. The area of a square field is 1.69 sq. m. Find the length of its one side?

   Ans: Area of a square = side × side = $(\text{side})^2$

   $\text{side} = \sqrt{\text{Area}} = \sqrt{1.69} = \mathbf{1.3m}$

3. Find out = $\sqrt{441} \times \sqrt{324}$ (L.I.C. Asst. Grade Exam)

   Ans: $\sqrt{441} \times \sqrt{324} = 21 \times 18 = \mathbf{378}$

4. Find the square root of $\sqrt{\frac{1.44}{0.0001}}$ (MBA Entrance Exam.)

$$\frac{\sqrt{1.44}}{\sqrt{0.0001}} = \frac{1.2}{0.01} = \frac{1.2 \times 100}{0.01 \times 100} = \frac{120}{1} = 120$$

5. Find the value of k in $\sqrt{\frac{484}{k}} + 14 = 25$ (S.B.I. P.O. Exam.)

Ans: $\frac{\sqrt{484}}{k} = 25 - 14 = 11$

$$22 = 11 \times k$$

$$\text{ie } 11k = 22$$

$$k = \frac{22}{11} = \mathbf{2}$$

6. If $\sqrt{324} \div \sqrt{x} = 9$. then x equals _____. (Clerks Grade Exam)

Ans: $\sqrt{324 \div x} = \mathbf{9}$

Squaring both sides

$$324 \div x = 9^2 = 81$$

$$324 = 81 \times x$$

$$\frac{324}{81} = x$$

$$4 = x$$

$$\therefore x = \mathbf{4}$$

7. $\frac{\sqrt{81}}{\sqrt{0.09}} = ?$ (R.R.B. Exam.)

Ans: $\frac{\sqrt{81}}{\sqrt{0.09}} = \frac{9}{0.3} = \frac{9 \times 10}{0.3 \times 10} = \frac{90}{3} = \mathbf{30}$

8. $\left(\sqrt{16}\right)^2 = ?$ (Bank P.O. Exam.)

Ans: $\left(\sqrt{16}\right)^2 = 16$. Hint. $\left(\sqrt{a}\right)^2 = a$

If a and b are perfect squares. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

If $a^2 = b$, then $a = \sqrt{b}$

$\sqrt[2]{a} = \sqrt{a}$. For square root no need of writing 2 as index.

C. CUBES AND CUBE ROOTS

$a^3 = a \times a \times a$

$\sqrt[3]{a^3} = a$

$\sqrt[3]{a}$ is called a radical where,

a is called a radicand and

3 is called the index of the radical.

If a and b are two integers,

$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$

$\sqrt[3]{a} = a^{1/3}$

a	a^3
1	1
2	8
3	27
4	64
5	125
6	216

The number 1729 is called Ramanujan's number. It is the smallest number which can be expressed as a sum of two cubes in two different ways.

$$12^3 + 1^3 \text{ and } 10^3 + 9^3$$

D. SURDS

Numbers which are not perfect squares are called surds.

Ex: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$ etc.

Square roots of 1,4,9,16,25 etc are whole numbers. Therefore, these are not surds.

Certain numbers can be written as the product of an integer and a surd.

Ex: $\sqrt{108} = \sqrt{2\times2\times3\times3\times3}$

$= \sqrt{2^2\times3^2\times3}$

$= 2\times3\times\sqrt{3}$

$= 6\sqrt{3}$

If the radical parts of two or more surds are equal, they are called like surds.

Ex: $3\sqrt{2}$ and $4\sqrt{2}$

$2\sqrt{3}$ and $5\sqrt{3}$

$7\sqrt{5}$ and $4\sqrt{5}$ etc.

The product of like surds will always be a rational number.

Ex: $\sqrt{2}\times\sqrt{2} = \sqrt{4} = 2$

$\sqrt{3}\times\sqrt{3} = \sqrt{9} = 3$

Problems:

1. $5\sqrt{2} + 3\sqrt{2} + \sqrt{2} = 9\sqrt{2}$

2. $8\sqrt{3} - 5\sqrt{3} = 3\sqrt{3}$

3. $4\sqrt{5}\times 3\sqrt{5} = 4\times3\times\sqrt{5}\times\sqrt{5} = 12\times5 = \mathbf{60}$

4. $\left[\sqrt{10}\times\sqrt{15}\right]$ is equal to _____. (Clerks Grade Exam)

Ans: $\sqrt{10}\times\sqrt{15} = \sqrt{10\times15} = \sqrt{150} = \sqrt{2\times3\times5\times5}$

$= \mathbf{5\sqrt{6}}$

5. $\dfrac{12}{\sqrt{2}} = \dfrac{12\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \dfrac{12\sqrt{2}}{2} = \mathbf{6\sqrt{2}}$

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14. SIMPLE INTEREST AND COMPOUND INTEREST

A. SIMPLE INTEREST

If interest is calculated on the original principal throughout the loan period, it is said to be simple interest. Interest on a given sum can be calculated by applying the idea of percent and unitary method.

$$I = P.N.R.$$
$$A = P + I$$

P = Principal, ie, the money borrowed from a lender.

N = Number of years

R = Rate of interest

I = Interest, ie, the additional money payable after a specified period inaddition to principal.

A = P + I. ie, total money payable at the end of a specified period.

Problems:

1. A sum of Rs. 800 is lent for 4 years at the rate of 12% per annum. Find the amount to be paid after the period ?

Ans:

$I = P\,N\,R$

$= \text{Rs. } 800 \times 4 \times \frac{12}{100}$

$= \text{Rs. } 384$

P = Rs.800
N = 4 years
R = 12% = $\frac{12}{100}$

Amount after 4 years $= P + I$

$= \text{Rs. } 800 + \text{Rs. } 384$

$= \mathbf{Rs.\ 1184}$

2. In how many years will a sum double itself at 12.5% simple interest per annum?

Ans: The sum doubles itself means P becomes 2P.

$\therefore$ Simple interest (I) = 2P - P = P

$P\,N\,R = I$

$P \times N \times \frac{12.5}{100} = P$

$N = \frac{P}{P} \times \frac{100}{12.5} = \mathbf{8}$ **years**

3) A sum deposited at a bank fetches Rs.13,440 after 5 years at 12% simple interest. Find the principal amount ?

(ICWA Preliminary Exam.)

Ans:

$P + I = A$

$P + \left(P \times 5 \times \frac{12}{100}\right) = \text{Rs. } 13{,}440$

$P + \frac{3}{5} P = \text{Rs. } 13{,}440$

$1\frac{3}{5} P = \frac{8}{5} P = \text{Rs.} 13{,}440$

Principal amount **P** = $\text{Rs.} 13.440 \times \frac{5}{8} = \mathbf{Rs.\ 8400}$

4. Find the present worth of Rs.651 due 3 years hence at 8% per annum simple interest ?

(Pre-Degree Commerce, Kerala University)

Ans:

$P + I = \text{Rs. } 651$

$P + PNR = \text{Rs.} 651$

$P + \left(P \times 3 \times \frac{8}{100}\right) = \text{Rs.} 651$

$1P + 0.24P = \text{Rs.} 651$

$1.24P = \text{Rs.} 651$

Present value: $P = \frac{\text{Rs.} 651}{1.24} = \mathbf{Rs.525}$

5. The simple interest on a sum of money at 5% is Rs.48 for 4 years. The simple interest on the same sum for 5 years at 4% will be _____. (C.D.S. Exam.)

Ans: PNR = I

$P \times 4 \times \frac{5}{100}$ = Rs.48

$P \times \frac{1}{5}$ = Rs.48

P = Rs. 48 × 5 = Rs.240

Simple interest on Rs. 240 for 5 years at 4% interest = $Rs.240 \times 5 \times \frac{4}{100}$ = **Rs.48**

Direct solution :

Since **P** being the same, $P \times 4 \times \frac{5}{100} = P \times 5 \times \frac{4}{100}$

∴ Interest in both cases is the same. ie, Rs.48

6. Rs.800 amounts to Rs.920 in three years at simple interest. If the interest rate is increased by 3%, it would amount to how much? *(Bank P.O., Exam)*

Ans: P N R = I

$Rs.800 \times 3 \times \frac{R}{100}$ = 920 - 800

Rs. 24 × R = Rs. 120

$R = \frac{Rs.120}{Rs.24} = 5\%$

Net interest rate = 5% + 3% = **8%**

Interest = $Rs.800 \times 3 \times \frac{8}{100}$ = Rs. 192

Total amount = Rs.800 + Rs.192 = **Rs.992**

B. COMPOUND INTEREST

FORMULAE :

S = A = sum of money after certain period

$S = P(1 + r)^n$

P = principal

Compound interest = $P(1+r)^n - P$

$r = \frac{R}{100}$ = *Rate of interest in decimal*

n = number of years

To find half yearly compound inerest :	$S = P(1 + \frac{r}{2})^{2n}$
To find quarterly compound interest :	$S = P(1 + \frac{r}{4})^{4n}$
To find monthly compound interest :	$S = P(1 + \frac{r}{12})^{12n}$

Depreciation:- written down value method :

$S = P(1 - r)^n$

Problems:

1. If principal is Rs.12,000 and rate of interest is 10%, find the Compound interest and amount after two years.

Ans: (a) Without applying formulae :

Principal for the first year (P_1) = Rs.12000

Interest for the first year (I_1) = $Rs. 12000 \times \frac{10}{100}$

= Rs. 1200

Principal for the second year (P_2) = Rs.12000 + Rs.1200

= Rs.13200

Interest for the second year (I_2) = $Rs.13200 \times \frac{10}{100}$

= Rs.1320

Amount after two years = Rs.13200 + Rs.1320

= **Rs.14520**

Interest for two years = $I_1 + I_2$

= Rs.1200 + Rs.1320

= **Rs.2520**

or

= Amount - Principal

" = Rs.14520 - Rs.12000

= **Rs.2520**

(b) Applying formulae :

$S = P(1+r)^n$

$= Rs.12,000\ (1+0.1)^2$ *P = Rs.12,000*

$= Rs.12000 \times (1.1)^2$

$= Rs.\ 12000 \times 1.21$ *r = 10%= $\frac{10}{100}$ = 0.1*

= **Rs.14520**

n = 2 years

Amount after two years = Rs.14520

Interest for two years = Rs.14520 - Rs.12000 = **Rs.2520**

2. The population of a certain village increases by 5% annually and its present population is 8000. The population after 3 years will be _____ . (M.B.A. Entrance Exam)

Ans: Apply the compound interest formulae.

$S = P(1 + r)^n$ *P = 8000*

$= 8000\ (1 + \frac{5}{100})^3$ *r = 5% = $\frac{5}{100}$*

n = 3 years

$= 8000\ (\frac{105}{100})^3$

$= 800(\frac{21}{20})^3$

$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} =$ **9261**

3. With compound interest, a sum of money amounts to Rs. 1452 in two years and Rs.1597.20 in 3 years. what is the rate of interest % per annum ? (R.R.B. Exam.)

Ans: Formula for compound interest $(S) = P(1+r)^n$

where, P = Principal, r = rate of interest and n = no. of years

Sum after 2 years $= P(1+r)^2 = 1452.00$ ____ (1)

Sum after 3 years $= P(1+r)^3 = 1597.20$ ____ (2)

$(2) \div [1]\ \frac{P(1+r)^2}{P(1+r)^2} = \frac{1597.20}{1452}$

$[1+r] = 1.1$

$r = 1.1 - 1 = 0.1$

Rate of interest $= 0.1 \times 100$

$= 10\%$

OR

Increase in amount = Rs.1597.20 - Rs.1452

= Rs.145.20

145.20 is 10% of Rs. 1452, Hence rate of interest is 10%.

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15. MENSURATION

(Length, Area and Volume)

A. Lines and Angles

B. Two Dimentional Figures
(Triangle Quadrilateral, Polygon and Circle)

C. Three Dimentional Figures
(Cuboid, Cone, Cylinder & Sphers)

Mensuration is the measurement of geometrical quantities, such as lengths of lines, areas of plane or curved surfaces, and volumes of solids.

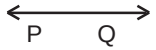
A. LINES AND ANGLES

Important Terms :

1. Point (ബിന്ദു) : . A = Point A

2. Line (രേഖ) :

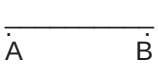
reads as line I.



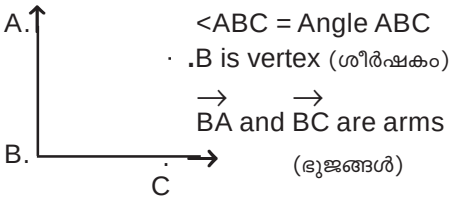
reads as line PQ.

3. Ray (രശ്മി) : 

, $\overrightarrow{AB}$ = Ray A B

4. Line Segment (രേഖാഖണ്ഡം) : 

, $\overline{AB}$ = Line segment AB

Angle (കോൺ) : 

<ABC = Angle ABC
.B is vertex (ശീർഷകം)
$\overrightarrow{BA}$ and $\overrightarrow{BC}$ are arms (ഭുജങ്ങൾ)

Instrument used to measure an angle is called Protractor.

Different Types of Angles:

a. Right angle (മട്ടകോൺ) : Its measure is 90^0

b. Accute angle (ന്യൂനകോൺ) : Its measure is between 0^0 and 90^0

c. Obtuse angle (ബ്രഹത്കോൺ) : Its measure is between 90^0 and 180^0

d. Straight angle : Its measure is 180^0

Complementary angles (പൂരക കോണുകൾ)

If the sum of the measures of two angles is 90^0. they are called complementary angles.

Supplementary angles (അനുപൂരക കോണുകൾ)

If the sum of the measures of two angles is 180^0, they are called Supplementary angles.

AREA AND VOLUME

Area (വിസ്തീർണ്ണം) : The magnitude of a plane region is called its area.

Volume (വ്യാപ്തം) : The magnitude of a space (solid) region is called its volume.

Perimeter or Circumference (ചുറ്റളവ്) : It is the length round the edge.

Application of Perimeter, Area and Volume

Cost of fencing around a field is calculated on the basis of perimeter.

Cost of a plot of land is calculated on the basis of area.

Cost of sand in a big box is calculated on the basis of volume.

Unit of length = metre = m

Unit of Area = sq.m. = m^2

Unit of volume = cu.m. = m^3

B. TWO DIMENSIONAL FIGURES

I. TRIANGLE (ത്രികോണം)

It has three sides and three angles. Sum of the measures of three angles is 180^0.

There are three types of triangles. They are

a. Equilateral Triangle (its 3 sides are equal),
b. Isosceles Triangle (its 2 sides are equal) and
c. Scalene Triangle (its 3 sides are different).

Formulae

Perimeter = Sum of the measures of three sides.

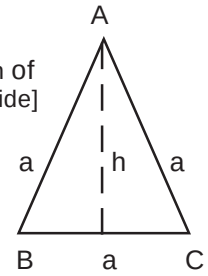
Area = ½ x one side x Perpendicular to that side.

1. Equilateral Triangle (സമഭുജ ത്രികോണം)

Area = $\frac{\sqrt{3}}{4} a^2$ [Where, a = length of one side]

Height = $\frac{\sqrt{3}}{2} a$

Perimeter = a + a + a = 3a

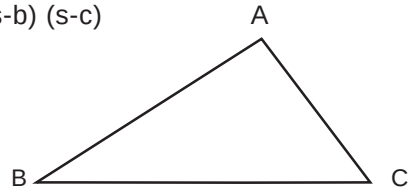


2. Scalene Triangle (വിഷമഭുജ ത്രികോണം)

Area = $\sqrt{s(s-a)\,(s-b)\,(s-c)}$

where, S = $\frac{a+b+c}{2}$

(ie, S=Semi-Perimeter and a, b, c are length of sides

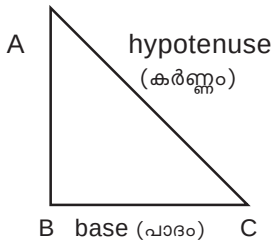


3. Right Triangle (മട്ടത്രികോണം)

Area = $\frac{1}{2}$ base ✕ height

$\frac{1}{2}$ **bh**

height or altitude (ലംബം)



Pythagoras' Theorem :

The Square of the hypotenuse of a right-angles triangle is equal to the sum of the squares of the other two sides.

$[\text{Hypotenuse}]^2 = [\text{base}]^2 + [\text{height}]^2$

$\text{Hypotenuse} = \sqrt{(\text{base})^2 + (\text{height})^2}$

$\text{Base} = \sqrt{(\text{hypotenuse})^2 - (\text{height})^2}$

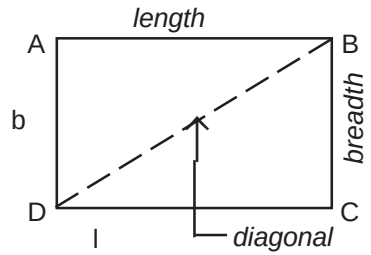
$\text{Height} = \sqrt{(\text{hypotenuse})^2 - (\text{base})^2}$

II. QUADRILATERAL (ചതുർഭുജങ്ങൾ)

It has four sides and four angles.
Sum of the measures of four angles is 360^0.

1. Rectangle (ചതുരം) :

Opposite sides of a rectangle are equal and parallel.
All angles are right angles.

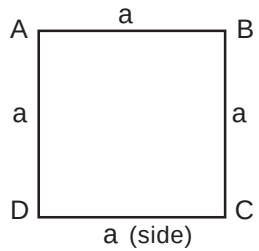


Area = Length x Breadth
= l ✕ b

Perimeter = 2l + 2b
= 2[l+b]

Diagonal = $\sqrt{l^2 + b^2}$

2. Square (സമചതുരം)



All sides of a square are equal and all angles of a square are equal in measure.

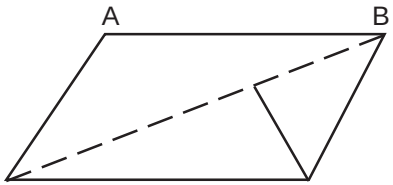
Area = a × a = a^2

Perimeter = a + a + a + a = 4a

Diagonal = $a\sqrt{2}$

Area of 4 walls = Perimeter of base × height

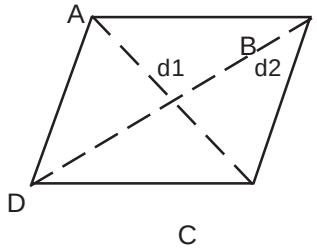
3. Parallelogram (സാമാന്തരികം)



Opposite sides and opposite angles of a parallelogram are equal. Opposite sides are parallel and diagonals bisect each other.

Area = Length of a diagonal × perpendicular distance from a vertex on the diagonal.

4. Rhombus

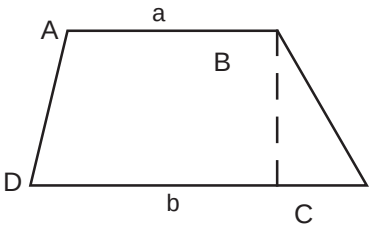


All sides of a rhombus are equal in length and opposite angles are equal.

Area = ½ × Product of two diagonals

$= \frac{1}{2}[d_1 \times d_2] = \frac{1}{2} \times d_1 \times d_2$

5. Trapezium



One pair of opposite sides of a trapezium are parallel.

Area = $\frac{1}{2}(a+b)h$

Where **a** and **b** are parallel sides and **h** is perpendicular distance between the parallel sides.

III. POLYGON

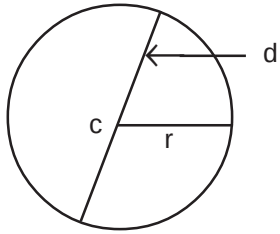
Polygon is a plane bounded by four or more straight lines. It is said to be regular when all sides are equal.

4 Sided polygon	:	Quadrilateral
5 sides polygon	:	Pentagon
6 sided polygon	:	Hexagon
7 sided polygon	:	Heptagon
8 sided polygon	:	Octagon

Number of diagonals of a polygon with 'n' sides = $\frac{n(n-3)}{2}$

Area of a Regular Hexagon = $\frac{6 \times \sqrt{3}\, a^2}{4}$

IV **CIRCLE** (വൃത്തം)



Middle point of a circle is Centre (c)
Distance between centre and circle is called radius (r)

Distance between two opposite points in a circle is called diameter (d)

d = 2r

Part of a circle is called **arc**.

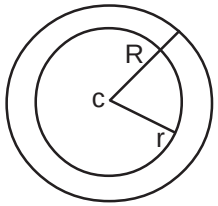
Formulae

Area = πr^2

Circumference = $2\pi r$

Length of an arc = $\frac{m}{360} \times 2\pi r$

$\frac{m\pi r}{180}$



Annulus of a circle = $\pi R^2 - \pi r^2$

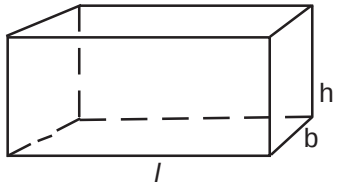
= $\pi[R^2 - r^2]$ or $\pi[R + r]\ [R - r]$

$\frac{\text{Circumference}}{\text{Diameter}}$ = π = $\frac{22}{7}$ = 3.14 (approx)

C. THREE DIMENSIONAL FIGURES

1. Cuboid or Rectangular Box (ചതുരപ്പെട്ടി)

It has 6 faces,
8 Vertices and
12 edges.



Volume = Length × Breadth × Height = **lbh**

Total surface Area = 2 [lb + bh + hl]

Length of Diagonal = $\sqrt{l^2 + b^2 + h^2}$

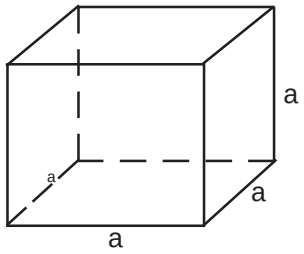
2. Cube

A Cuboid whose length, breadth and height are all equal is called a cube.

Volume = $(Side)^3 = a^3$

One edge = $\sqrt[3]{\text{Volume}}$

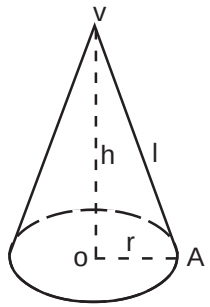
Total surface area = $6a^2$



Length of diagonal = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3}$

3. Right Circular Cone or Right Pyramid

Volume = $\frac{1}{3}\pi r^2 h$



Total Surface Area = $\frac{1}{2}$ × perimeter of base × slant height

= $\frac{1}{2} \times 2\pi r \times l = \pi rl$

Slant height [l] = $\sqrt{h^2 + r^2}$

4. Cylinder - Right Circular Cylinder (വൃത്തസ്തംഭം)

Volume = Base Area × height

= $\pi r^2 \times h$

= $\pi r^2 h$

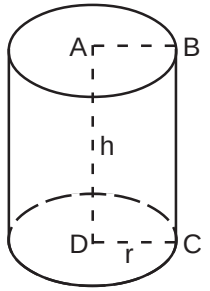
Area of Curved Surface = Circumference × height

= $2\pi r \times h$

= $2\pi rh$

Total Surface Area = $2\pi rh + 2\pi r^2$

= $2\pi r(h+r)$



5. Hollow right circular cylinder:

Volume = $\pi Rr^2h - \pi r^2h = \pi h[R^2 - r^2]$

Base Area = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Area of outer curved surface = $2\pi Rh$

Area of inner curved surface = $2\pi rh$

Total surface Area = $2\pi Rh + 2\pi rh + 2(\pi R^2 - \pi r^2)$

= $2\pi h(R + r) + 2\pi(R^2 - r^2)$

6. Sphere (ഗോളം)

Volume $= \frac{4}{3}\pi r^3$

Area $= 4\pi r^2$

7. Hemi - Sphere (അർദ്ധഗോളം)

Volume $= \frac{2}{3}\pi r^3$

Area $= \pi r^2 + 2\pi r^2 = 3\pi r^2$

8. Hollow Sphere

Volume = External volume - Internal volume

$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (R^3 - r^3)$

9. Prism

Volume of a Right Prism

= Area of base × Perpendicular height between the two ends.

Lateral surface Area = Perimeter of the base × height

Total surface Area = Lateral surface Area + 2 (Area of base)

Problems:

1) The base area of a cone is 12 cm2 and height is 8 cm, The volume of the cone is --------- cm3.

(S.S.L.C. Exam)

Ans: Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 12 cm^2 \times 8 cm = \mathbf{32 cm^3}$

2. A carpet of 16 metres breadth and 20 metres length was purchased at a cost of Rs. 2496. What is the cost per sq. metre ?

(Bank P.O. Exam)

Ans: Area of carpet = length × breadth

= 20 m × 16 m = 320 sq.m.

Cost of the carpet = Rs. 2496

Cost per sq.metre $= \frac{\text{Rs. 2496}}{\text{320 sq.m.}} = \mathbf{Rs.7.80}$

3. If the volume and surface area of a sphere are numerically the same, then its radius is _____ .

(N.D.A. Exam.)

Ans: Let 'r' be the radius of the sphere.

Then, its volume = its surface area

$\frac{4}{3}\pi r^3 = 4\pi r^2$

$r^2 = r \times r \times r$

$= r^2 \times r$

$\frac{4}{3} \times \pi \times r^2 \times r = 4 \times \pi \times r^2$

Cancel πr^2 from both sides

Then, $\frac{4}{3} r = 4$

$r = 4 \times \frac{3}{4} = 3$

Radius = **3 units**

4. The perimeter of a garden is 140 metres and its sides are 4:3 ratio. Half the area is planted with roses. If each square metre of area is having 5 rose plants, how many rose plants are there ?

(L.I.C. Asst. Grade Exam.)

Ans: Let the length and breadth of the garden be 4x metres and 3x metres respectively.

Perimeter : 2[l + b] = 140 m

$2(4x + 3x) = 140$

$7x = \frac{140m}{2} = 70\ m$

$x = \frac{70\ m}{7} = 10\ m$

Length (l) = 4x = 4 x 10 = 40

Breadth (b) = 3x = 3 x 10 = 30

Total area [l × b] = 40m × 30m = 1200sq.m.

Number of rose plants $= \frac{1200\ sq.m.}{2} \times 5$ nos/sq.m.

= **3000 numbers**

5. How many liters of water can a hemispherical vessel whose diameter is 20 cm contain ?

[L.I.C. Asst. Grade II]

Ans:

Diameter of the vessel = 20 cm, ∴ radius (r) $= \frac{20}{2} = 10$ cm.

Volume of a hemi-sphere $= \frac{2}{3}\pi r^3$

Volume of the vessel $= \frac{2}{3} \times \frac{22}{7} \times (10)^3$

$= \frac{44}{21} \times 1000 = 2095 cm^3$

Capacity of the vessel $= \frac{2095\ cm^3}{1000} =$ **2.095 litres**

(Hint: 1 litre water = 1000 cm^3)

6) The angles in a quadrilateral are b, (b+30), (b+40), (b+50). The smallest angle is _____.

[R.R.B. Exam]

Sum of the measures of a quadrilateral = 360^0

Ans: (b) + (b +30)+(b+40)+(b+50) = 360^0

$4b+120 = 360^0$

$4b = 360 - 120 = 240^0$

smallest angle → b $= \frac{240}{4} =$ **60^0**

7) If the diameter of a circle is increased by 100%, its area is increased by _____.

(Admission Test to Deploma in Hotel Management)

Ans: Radius (r) is half of diameter.

Area = πr^2

New radius = 2r

∴ New area = $\pi[2r]^2 = \pi \times 4 \times r^2 = 4\pi r^2$

∴ Increase in area = $4\pi r^2 - \pi r^2 = 3\pi r^2$

Thus, the area increases 300%, ie, **3 times.**

8) The length and breadth of a square are increased by 10% and 5% respectively. Then the area of the resulting rectangle exceeds the area of the sqaure by what percentage.?

(P S C Divisional Accountant Test)

Ans: Let the side of square be *X* units.

Then, Area = $X \times X = X^2$ sq.units.

Length and breadth after increase = 1.1*X* and 1.05*X* respectively

Area of resulting rectangle = $1.1X \times 1.05X$

$= 1.155X^2$

Excess area = $1.155X^2 - 1X^2$

$= 0.155X^2$

Excess in percentage = 0.155 × 100 = **15.5%**

9) Find the total surface area of the cube whose volume is 125 cm^3. (I.C.W.A. Preliminary Exam)

Ans: Total surface area of a cube = $6a^2$

Volume of a cube = a^3 = 125

$$a = \sqrt[3]{125} = 5$$

Total surface area of the cube = $6 \times 5 \times 5$

= **150 cm^2**

10) How many solid spheres each of 12 cms. diameter could be moulded from a solid metal cylinder whose height is 72 cms. and diameter 8 cms ? [I.C.W.A Intermediate Exam]

Ans: Height of the solid metal cylinder (h) = 72 cms.

Radius of the solid metal cylinder (r) = $\frac{8}{2}$ = 4cms

Volume of metal cylinder = $\pi r^2 h$

= $\pi \times 4^2 \times 72$

= $\pi \times 16 \times 72$

= 1152π cm^3

Diameter of solid sphere = 12 cm

Radius (r) = $\frac{12}{2}$ = 6 cm

Volume of each sphere = $\frac{4}{3} \times \pi \times 6 \times 6 \times 6$

= 288π cm^3

$\therefore$ Number of spheres that can be moulded from the cylinder = $\frac{\text{Total volume of metal cylinder}}{\text{volume of each sphere}}$

= $\frac{1152\pi \text{ cm}^3}{288\pi \text{ cm}^3}$ = **4**

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16. MISCELLANEOUS CHAPTERS

A) ARITHMETIC AND GEOMETRIC PROGRESSION

Arithmetic Progression (A.P.)

A series of quantities is said to be in A.P. when the quantities increase or decrease continually by a common quantity.

Let us denote the first, second, third, ..., n^{th} term of an A.P. by t_1, t_2, t_3, ..., $t_n - 1$, t_n

Common difference (d) of an A.P. =

Any term of the A.P. – the term just preceeding it.

$d = t_2 - t_1 = t_3 - t_2 = t_n - (t_n - 1)$

If a, b, c are three consecutive terms of an A.P.,

then b - a = c - b

ie. 2b = a + c

a = first term

n = no. of terms

d = common difference

Formulae

1. n^{th} term of an A.P. :
$$t_n = a + (n-1)d$$
2. Sum of first n terms of an A.P. :
$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$
3. Number of terms in a finite A.P. :
$$n = \frac{\text{(Last term - First term)}}{\text{Common difference}} + 1$$
4. Sum of integers between two numbers :
$$S = \frac{n}{2}\text{(First term + Last term)}$$
5. Sum of 'n' counting numbers starting from one :
$$= \frac{n(n+1)}{2}$$

Problems:

1. Find 12^{th} term of an A.P., if a = 3 and d = 4 ?

Ans: n^{th} term $= a + (n-1)d$

12^{th} term $= 3 + (12 - 1)4$

$= 3 + 11 \times 4 = 3 + 44 =$ **47**

2. Find the sum of 8 terms beginning from 13 and its common difference is 5 ? S.S.L.C Exam)

Ans: Sum of **n** numbers $= \frac{n}{2}\{2a + (n-1)d\}$

Sum of **8** numbers $= \frac{8}{2}\{2\times13 + (8-1)5\}$

$= 4(26 + 7\times5)$

$= 4(26 + 35) = 4\times61 =$ **244**

3. Find the sum of first 200 natural numbers ?

(P.S.C. L.D. Clerk Exam)

Ans: $S_n = \frac{n(n+1)}{2}$

$S_{200} = \frac{200(200+1)}{2} = \frac{200 \times 201}{2} = \frac{40200}{2} =$ **20100**

4. 1+2+3+4+5.... .... + 100 = ...

(Clerks Grade Exam)

Ans: $S_{100} = \frac{100(100+1)}{2} = \frac{100 \times 101}{2} = \frac{10100}{2} =$ **5050**

5. The sum of the numbers 1+2+3+4+5+.....+99 is equal to _____.

(Deploma in Hotel Management Entrance Exam)

Ans: $S_{99} = \frac{99 \times 100}{2} = \frac{9900}{2} =$ **4950**

6. Find the sum of all numbers between 71 and 771 that are divisible by 7. (Pre-Degree (Maths) 1st year (MGU)

Ans: $S_n = \frac{n}{2}$ (First term + Last term)

$7\overline{)71}\ 10$, 7, 1

First term which is divisible by 7 = 77

(ie. next multiple of 7 after 71)

Last term which is divisible by 7 = 770

(i.e., Multiple of 7 just preceding 771)

Number of terms (n) $= \frac{\text{(Last term - First term)}}{\text{Common difference}} + 1$

$= \frac{(770 - 77)}{7} + 1 = \frac{693}{7} + 1$

$= 99 + 1 = 100$

Sum (S_n) $= \frac{n}{2}$ (First term + Last term)

$= \frac{100}{2}$ [77 + 770]

$= 50 \times 847 =$ **42350**

Geometric Progression (G.P.)

G.P. is a sequence in which each term after the first term is got by multiplying the just preceding term by the same fixed number. The fixed number is called Common Ratio (C.R.) which is usually denoted by the letter 'r'.

If we denote $t_1, t_2, t_3 \ldots\ldots\ldots t_n\text{-}1, t_n$ are terms of a G.P.,

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}} = \frac{\text{Any term of the G.P.}}{\text{Term just preceeding it.}}$$

If a, b, c are three consecutive terms of a G.P., then

$\frac{b}{a} = \frac{c}{b}$, ie. $b^2 = a\,c$

Formulae

1. n^{th} term of a G.P. $= a.\ r^{n-1}$
2. Sum of n terms of a G.P. :-

$$S_n = \frac{a(r^n - 1)}{r-1}, \text{ When } r > 1$$

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ When } r < 1$$

Problems:

1. The 3rd and 6th terms of a series in GP are 3 and 81 respectively, find the first term and common ratio ?

Ans: n^{th} terms of a G.P. $= a.\ r^{n-1}$ — a = first term

Here $t_3 = ar^2 = 3$ — r = common ratio

$t_6 = ar^5 = 81$ — $ar^2 = a \times r^2$

$$\therefore \frac{ar^5}{ar^2} = \frac{81}{3}$$

$$r^3 = 27 = 3^3$$

$$\therefore r = 3$$

Again, $ar^2 = 3$

$$a \times 9 = 3$$

$$a = \frac{3}{9} = \frac{\mathbf{1}}{\mathbf{3}}$$

2. The first and last terms of a G.P. are 3 and 768 respectively and the sum is 1533. Find the common ratio and the number of terms.

$t_1 = a = 3$

Suppose $t_n = n^{th}$ term $= 768$

$t_n = a.\ r^{n-1} = 768$

$3 \times r^{n-1} = 768$

$r^{n-1} = 768 \div 3 = 256$ ---- (1)

$$S_n = \frac{a(r^n-1)}{r-1} = 1533$$

$$\frac{r^n - 1}{r-1} = \frac{1533}{3} = 511$$

$$\frac{r^n - 1}{r-1} = \frac{1533}{3} = 511$$

$r^n - 1 = 511 \times (r-1)$

$r^n = 511\,(r-1)+1$ ---- (2)

Dividing (2) by (1) $\frac{r^n}{r^{n-1}} = \frac{511\,(r-1)+1}{256}$

$$r = \frac{511\,(r-1)+1}{256}$$

$256\,r = 511r - 511 + 1$

$256r - 511r = -510$

$-255r = -510$

Common ratio: $r = \frac{-510}{-255} = +2$

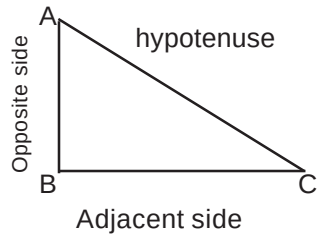
Putting the value of **r** in (1), $2^{n-1} = 256 = 2^8$

$n-1 = 8$

Number of terms : $n = 8 + 1 = \mathbf{9}$

B) TRIGONOMETRY

Trigonometry is a branch of mathematics dealing with measures of triangles. It is used to calculate heights, distances, areas etc. which could not be directly measured. For example we can calculate heights of mountains, height at which an aeroplane was flying at a particular point of time, distance between stars, etc with the help of trigonometry. We use θ (theta) to denote the measure of an angle of a triangle.



$$\text{Sin } \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Cos } \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\text{Tan } \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\text{Sin } \theta}{\text{Cos } \theta}$$

$$\text{Cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{1}{\text{Sin } \theta}$$

$$\text{Sec } \theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{1}{\text{Cos } \theta}$$

$$\text{Cot } \theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{1}{\tan \theta}$$

TRIGONOMETRIC RATIOS

Name of Ratio	0^0	30^0 $\left(\frac{\pi}{6}\right)$	45^0 $\left(\frac{\pi}{4}\right)$	60^0 $\left(\frac{\pi}{3}\right)$	90^0 $\left(\frac{\pi}{2}\right)$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	α
Cosec	α	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	α
Cot	α	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

TRIGONOMETRIC IDENTITIES

$$\text{Sin}(90 - \theta) = \text{Cos } \theta$$

$$\text{Cos}(90 - \theta) = \text{Sin } \theta$$

$$\text{Tan}(90 - \theta) = \text{Cot } \theta$$

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$\text{Sec}^2\theta - \tan^2\theta = 1$$

$$\text{Cosec}^2\theta - \text{Cot}^2\theta = 1$$

C) ROMAN NUMBERS

The ancient Romans invented a system of numeral for counting. It became known as the Roman Numbers. There are only seven numerals in this system.

DIGIT	
I = 1	$\overline{I}$ = 1 x 1000 = 1000
V = 5	$\overline{V}$ = 5 x 1000 = 5000
X = 10	$\overline{XI}$ = 11 x 1000 = 11000
L = 50	$\overline{LV}$ = 55 x 1000 = 55000
C = 100	$\overline{IC}$ = 99 x 1000 = 99000
D = 500	$\overline{D}$ = 500 x 1000 = 500000
M = 1000	$\overline{M}$ = 1000 x 1000 = 1000000

Any digit can be added to another one subject to the maximum of 3 times.

eg. 6 = 5 + 1 = V + I = VI

12 = 10 + 2 = X + II = XII

53 = 50 + 3 = L + III = LIII

Any digit except V, L and D can be subtracted from a digit only one time.

eg: 4 = 5 -1 = V - I = IV

9 = 10-1 = X - I = IX

45 = 50-10+5 = L-X+V = XLV

If a numeral of a lesser value is written after a numeral of a greater value, the resulting number is obtained by adding their value. If a numeral of a lesser value is written before a numeral of a greater value, the resulting number is obtained by finding their difference.

The basic numeral **V** is never subtracted.

I can be subtracted from **V** and **X** only.

CONVERSION

From Whole number to Roman number		From Roman number to Whole number	
88	LXXXVIII	CML	950
89	XIC	MDCLXVI	1666
90	XC	DXLVII	547
91	XCI	$\overline{XC}$VI	96
901	C$\overline{M}$I	$\overline{IX}$	9000
5001	$\overline{V}$ I	$\overline{IX}$	9999
11111	XI CXI	V CIX	5109

eg: Roman number VDCXIV is equal to _____.

(Deploma in Hotel Management Entrance Exam)

Ans: VDCXIV = 5000 + 600 + 14 = **5614**

D) BINARY NUMBERS

Binary number system is used to represent data in a computer. The advantage of this number system is that there are only two digits 0 and 1.

Decimal System

In decimal system, there are 10 unique digits (0,1,2,....., 9). Since it has ten states, it is known as base 10 system.

10^4	10^3	10^2	10^1	10^0	Exponential value of position
10000	1000	100	10	1	Integer value of position

eg: The number 1234 in the decimal system is represented by :

From right to left :

$4 \times 10^0 = 4 \times 1 = 4$

$3 \times 10^1 = 3 \times 10 = 30$

$2 \times 10^2 = 2 \times 100 = 200$

$1 \times 10^3 = 1 \times 1000 = 1000$

1234

===

Binary System

2^5	2^4	2^3	2^2	2^1	2^0	Exponential value of position
32	16	8	4	2	1	Integer value of position

eg: 1101_{two} can be written in decimal system as :

From right to left :

$1 \times 2^0 = 1 \times 1 = 1$
$0 \times 2^1 = 0 \times 2 = 0$
$1 \times 2^2 = 1 \times 4 = 4$
$1 \times 2^3 = 1 \times 8 = 8$
13

$1101_{two} = 13_{ten}$

Probems :

1. Convert $(1110111)_2$ into base 10 system.

[ICWA Final Exam]

Ans :

From right to left

$(1110111)_2 = (119)_{10}$

$1 \times 2^0 = 1 \times 1 = 1$
$1 \times 2^1 = 1 \times 2 = 2$
$1 \times 2^2 = 1 \times 4 = 4$
$0 \times 2^3 = 0 \times 8 = 0$
$1 \times 2^4 = 1 \times 16 = 16$
$1 \times 2^5 = 1 \times 32 = 32$
$1 \times 2^6 = 1 \times 64 = 64$
119

2. Convert $(43)_{10}$ into base two system.

Ans:

2	43		*Remainders*
2	21	→ 1	
2	10	→ 1	**$(43)_{10} = (101011)_2$**
2	5	→ 0	
2	2	→ 1	
2	1	→ 0	Binary equivalent starts from last remainder.
2	0	→ 1↑	

Addition

0 + 0 = 0
0 + 1 = 1
1 + 1 = 10
1 + 1 + 1 = 11

Ex: Binary	Decimal
101011	43
110010	50
1011101	**93**

Subtraction

0 - 0 = 0
1 - 0 = 1
1 - 1 = 0
10 - 1 = 1

Ex: Binary	Decimal
1011	11
0111	7
100	**4**

Multiplication

Ex: Binary	Decimal
101	5 ×
11	3
101	**15**
101	
1111	

Division

Ex: Binary	Decimal
1001	9
101 | 101101	5 | 45
101	45
101	**00**
101	
000	

Computer Memory Capacity

1 Kilobyte (KB)	=	1024 bytes
1 Megabyte (MB)	=	1024 KB
1 Gigabyte (GB)	=	1024 MB
1 Terrabyte (TB)	=	1024 GB
1 Petabyte (PB)	=	1024 TB
1 Exabyte (EB)	=	1024 PB
1 Zettabyte (ZB)	=	1024 EB
1 Yottabyte (YB)	=	1024 ZB

E) CENTIGRADE AND FAHRENHEIT

These scales are used to measure temperature. Freezing point of water is 0^0 C or 32^0F and boiling point is 100^0C or 212^0 F.

Formula for conversion :

$$\mathbf{C = \frac{5}{9}\ (F-32)}$$

C = Centigrade or Celsius
F = Fahrenheit

Ex: 1. The normal temperature of a healthy person is 98.4^0F. Express this in celsius scale.

Ans:

$$C = \frac{5}{9}\ (F-32)$$

$$= \frac{5}{9}\ [98.4-32]$$

$$= \frac{5}{9} \times 66.4$$

$$= \frac{332}{9}$$

$$= 36.9^0\ c$$

$$\mathbf{98.4^0F = 36.9^0\ c} \quad \text{(approximately)}$$

2. Convert -4^0c into ^{0}F. (ITI Two year Final Exam)

Ans :

$$\frac{5}{9}\ (F - 32) = C$$

$$\frac{5}{9}\ (F - 32) = -4$$

$$F - 32 = -4 \times \frac{9}{5} = \frac{-36}{5} = -7.2$$

$$F = -7.2 + 32 = +24.8$$

$$\mathbf{-4^0c = 24.8^0F}$$

F) UNITS OF MEASUREMENT

Length	:	Metre (m.)
Perimeter	:	metre (m.)
Area	:	Square metre (sq.m.)
Volume	:	Cubic metre (cu.m.)
Time	:	Second (sec.)
Weight	:	Kilo gram (kg.)
Liquid	:	Litre (l)
Electricity	:	Ampere (A)
Electric Power	:	Watt (W)
Power of Machines	:	Horse Power (H.P.)

'Kilo' means 1000, Kilo metre means 1000 metres

'Centi' means $\frac{1}{100}$, Centi metre means $\frac{1}{100}$ of metre

'Milli' means $\frac{1}{1000}$, Milli metre means $\frac{1}{1000}$ of metre

SYSTEM OF UNITS

Unit	C.G.S. System [Metric system]	F.P.S. System British system	M.K.S. System (Metric system)
Length	Centimetre	Foot	Metre
Mass	Gram	Pound	Kilogram
Time	Second	Second	Second

G) CONVERSION TABLES

LENGTH

1 cm.	=	10 mm.		
1 deci m.	=	10 cm.		
1 m.	=	10 deci m.	=	100 cm.
1 deka m.	=	10 m.		
1 hecto m.	=	10 deka m.	=	100 m.
1 km.	=	10 hecto m.	=	1000 m.
1 inch	=	2.54 cm.		
1 foot	=	12 inches	=	30.48 cm
1 metre	=	3.2808 feet	=	39.37 inches
1 metre	=	1.0936 yards		
1 mile	=	1.6093 km.		

AREA

1 Sq. inch	=	6.452 sq. cm.		
1 Sq. foot	=	929 sq.cm.	=	144 sq. inch
1 Sq. m.	=	10.764 sq. feet	=	1.196 sq.yards
1 Cent	=	40.47 sq.m.	=	1000 sq.links
1 Are	=	100 sq.m.	=	2.471 cents.
1 Hectar	=	2.471 Acres		
1 Sq. mile	=	2.589 sq. km.		

VOLUME

1 Cu. inch	=	16.387 cu.cm.		
1 Cu. m.	=	35.315 cu. feet	=	1.3080 cu. yard

MASS

1 ounce	=	28.35 grams
1 Kg	=	2.2046 lbs
1 quintal	=	100 kg
1 Tonne (MT)	=	1000 kg

CAPACITY

1 milli litre	=	1 cu. cm.
1 litre	=	1000 cu. cm.
1 kilo litre	=	1 cu. m.
1 Pint	=	568 milli litre
1 gallon	=	4.54609 litres

1”	=	1 inch
1‘	=	1 foot
1 lb	=	1 Pound

1 m. = 100 cm

$1\ m^2$ = 100 cm × 100 cm = 10000 cm^2

$1\ m^3$ = 100 cm × 100cm × 100cm
= 1000000 cm^3

Note:

1 mile = 1.6093 km

$1\ km. = \frac{1}{1.6093}\ mile = 0.6214\ mile$

Likewise you can convert other units also.

Ex. 1 Are = 2.471 Cents, 1 Cent = $\frac{1}{2.471}$ Are = 0.4047 Are

H) LIST OF SYMBOLS

Addition	+
Subtraction	−
Multiplication	×
Division	÷
Is equal to	=
Is not equal to	≠
Is less than	<
Is not less than	<
Is greater than	>
Is not greater than	>
Is less than or equal to	≤
Is greater than or equal to	≥
Is an element of	∈
Is not an element of	∉
Is a subset of	⊂
Is not a subset of	⊄
Is a superset of	⊃
Is not a superset of	⊃
Is parallel to	//
Is perpendicular to	⊥
Is corresponds to	↔
Is congruent to	≅
Union	∪
Intersection	∩
Square root	√
Percent	%
Angle	∠
Triangle	Δ

I) MULTIPLICATION TABLE

(11 TO 20)

X	11	12	13	14	15	16	17	18	19	20
1	11	12	13	14	15	16	17	18	19	20
2	22	24	26	28	30	32	34	36	38	40
3	33	36	39	42	45	48	51	54	57	60
4	44	48	52	56	60	64	68	72	76	80
5	55	60	65	70	75	80	85	90	95	100
6	66	72	78	84	90	96	102	108	114	120
7	77	84	91	98	105	112	119	126	133	140
8	88	96	104	112	120	128	136	144	152	160
9	99	108	117	126	135	**144**	153	162	171	180
10	110	120	130	140	150	160	170	180	190	200
11	**121**	132	143	154	165	176	187	198	209	220
12	132	**144**	156	168	180	192	204	216	228	240
13	143	156	**169**	182	195	208	221	234	247	260
14	154	168	182	**196**	210	224	238	252	266	280
15	165	180	195	210	**225**	240	255	270	285	300
16	176	192	208	224	240	**256**	272	288	304	320
17	187	204	221	238	255	272	**289**	306	323	340
18	198	216	234	252	270	288	306	**324**	342	360
19	209	228	247	266	285	304	323	342	**361**	380
20	220	240	260	280	300	320	340	360	380	**400**

Example : 9 x 16 = 144